16. ## Capacitors:

Capacitor is a device which is used to store charges in the form of electrostatic energy

A capacitor is connected to battery. When pd across capacitor increases, the charge stored on it also increases $Q \alpha V$

Q=CV

C →Capacitance of capacitor

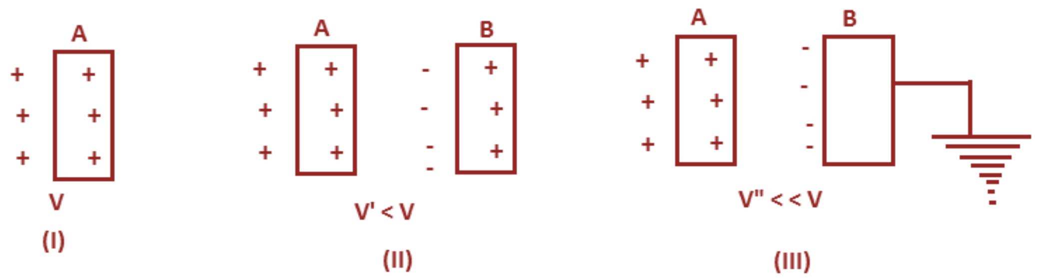
Define Capacitance of capacitor

Capacitance is defined as the ratio of amount of a charge stored in capacitor to the potential across plates of capacitor. C=Q/V

SI unit :

Capacitance of capacitor is said to be 1 farad when one coulomb of charge stored on it to raise is potential by 1V

17. ## Principle of capacitor :

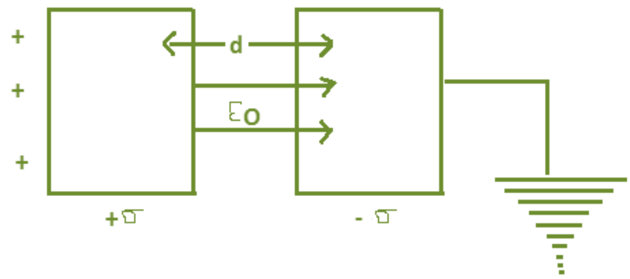


Consider a metal plate A carrying a charge Q (+ve) and potential raises to V. Now ,an uncharged metal plate B is placed nearer to A. –ve charge gets induced on B at the near of A and +ve charge is induced on other end as shown in fig(ii). Potential of A decreases by small amount due to induced charge on B. in order to get same potential, small amount of charge can be stored additionally.

When one end of metal plate B is earthed as in (iii) induced +ve charges are neutralized. Due to only Induced –ve charges, potential of A decreases by large amount. In order to get same potential, large amount of charge can be stored additionally.

Therefore, by placing an identical uncharged earthed metal plate , nearer to charged plate, capacitance of conductor can be increased.

18. <u>Capacitance of parallel plate capacitor filled with air</u>



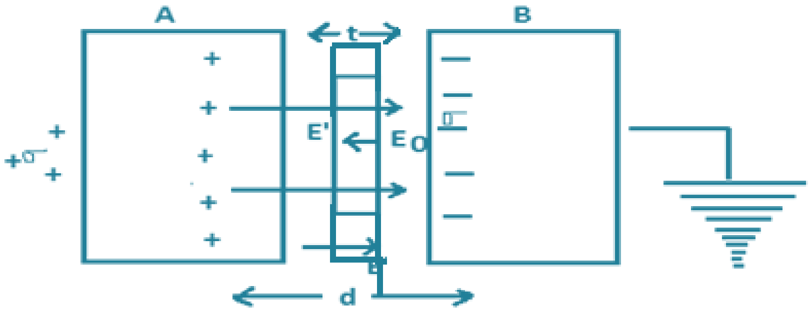
Consider two parallel plates Carrying charge density σ and – σ separated by distance d

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

$$V_0 = E_0 d = \frac{\sigma}{\varepsilon_0} d$$

$$C = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d} = \frac{A\varepsilon_0}{d}$$

19. <u>Capacitance of parallel plate capacitor partly filled with dielectric</u>



Area of each plate is A and one end of metal plates is earthed

Consider 2 parallel plates having charge density $+\sigma, -\sigma$ separated by a distance d. A dielectric slab of thickness $t < d$ is placed between the plates of capacitor. In the presence of $\vec{E}$, dielectric is Polarized. Net $\vec{E}$ is the difference of in E_o in air and

E 'induced charges on each plates; Net Electric field $E'' = E_o - E'$

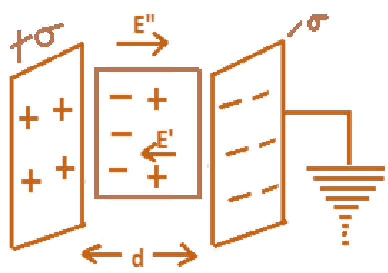
$$E_0 = \frac{\sigma}{\varepsilon_0} \; ; E'' = \frac{\sigma}{\varepsilon_0 \varepsilon_r}$$

$$V_0 = E_0(d-t) + E''t = \frac{\sigma}{\varepsilon_0}(d-t) + \frac{\sigma}{\varepsilon_0 \varepsilon_r}(t)$$

$$C = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}(d-t) + \frac{\sigma}{\varepsilon_0 \varepsilon_r}(t)} = \frac{A\varepsilon_0}{(d-t) + \frac{t}{\varepsilon_r}}$$

Capacitance increases

20 **<u>Obtain exp for capacitance of capacitor fully filled with dielectric</u>**



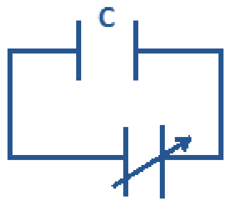
$$E'' = \frac{\sigma}{\varepsilon_0 \varepsilon_r} \; ; V'' = E'' d ; V'' = \frac{\sigma}{\varepsilon_0 \varepsilon_r} d$$

$$C_m = Q/V'' = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0 \varepsilon_r} d}$$

$$C_m = \frac{A\varepsilon_0\varepsilon_r}{d}$$

Capacitance increases by ε_r times

21 Energy stored in a capacitor



Consider uncharged capacitor of capacitance c connected to battery. Let q be charge stored on capacitor at an instant.

Potential developed $V' = q/c$

Small WD in storing charge dq additionally at the same potential V',

$(dW) = V'\,dq$

$dW = q/c\,dq$

Total WD in storing charges from O to Q.

$$W = \int_0^Q \frac{q}{C} dq = [\frac{q^2}{2C}]_0^Q = [\frac{Q^2}{2C}]$$

$$U = [\frac{Q^2}{2C}]$$

Sub $Q=CV; U=\frac{1}{2}CV^2$ Where $C=(A\varepsilon_0)/d$

Sub $CV=Q$

$(U=1/2\ QV)$

.

This WD is stored as electro static energy in the capacitor

22 **Energy density** u is defined as energy stored in the capacitor per unit volume

u= U/Vol=(1/2 CV^2)/Ad

$$=\frac{1}{2}\ (\frac{A\varepsilon_0}{d})\frac{E^2d^2}{Ad}=\frac{1}{2}\ \varepsilon_0 E^2$$

Obtain exp for energy stored in capacitor fully filled with dielectric and energy density

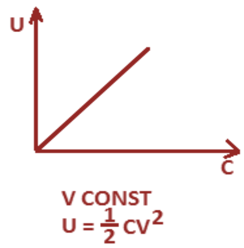
Derivation of energy stored. (as before)

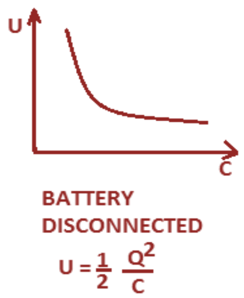
$$U=\frac{1}{2}CV^2\ where\ C=\frac{A\varepsilon_0\varepsilon_r}{d}$$

u= U/Vol=(1/2 CV^2)/Ad

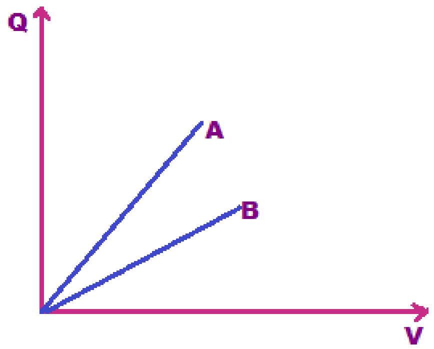
$$=\frac{1}{2}\ (\frac{A\varepsilon_0\varepsilon_r}{d})\frac{E^2d^2}{Ad}=\frac{1}{2}\ \varepsilon_0\varepsilon_r E^2$$

23. Battery is connected Battery is disconnected





24. Which capacitor has capacitance? Greater energy stored?



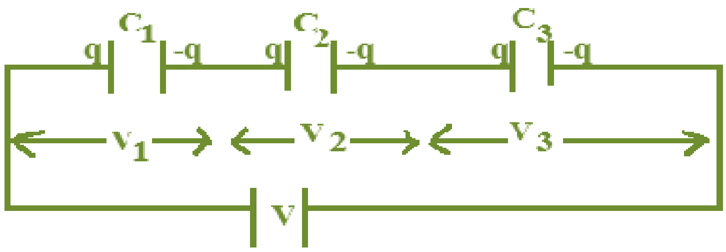
C=Q/V , slope of the graph gives capacitance

A has greater capacitance

Energy = Area under graph. A has greater energy stored.

Define dielectric constant in terms of capacitance $K = C_m/C_a$. Dielectric constant of the medium is defined as the capacitance of the parallel plate capacitor fully filled with medium to the capacitance of identical capacitor filled with air

25. Effective capacitance of capacitors connected in series.



Let C_1,C_2 andC_3 be capacitance of three capacitors connected in series. Battery has potential v. In series charge stored across each capacitor is same, while potential difference is different.

We know,

$Q=CV$

$Q=C_1V_1$,

$V_1=\frac{Q}{C_1}$,$V_2=\frac{Q}{C_2}$,$V_3=\frac{Q}{C_3}$

$V=Q/Ceff$

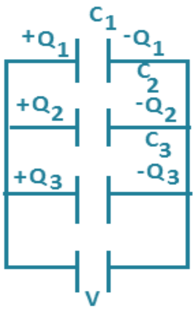
In series $V=V_1+V_2+V_3$

$Q/C_{eff} = Q[1/C_1 + 1/C_2 + 1/C_3]$

$\frac{1}{C_{eff}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$

Reciprocal of effective capacitance of capacitors in series in the sum of reciprocal of capacitance of individual capacitors.

26. Effective capacitance of capacitors to parallel.



Let C_1,C_2 andC_3be the capacitance of 3 capacitors connected in parallel. Battery has potential V. In parallel combination, charge stored across each capacitor is different and potential diff across each capacitor is same.

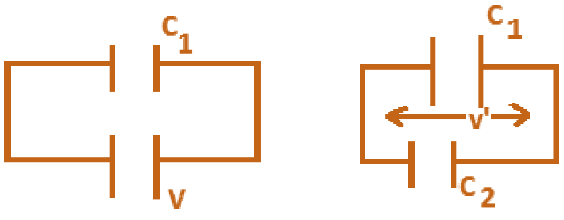
$Q=CV$; $V=V_1= V_2= V_3$;$Q_1=C_1V$; $Q_2=C_2V$; $Q_3=C_3V$; $Q=Q_1+ Q_2+ Q_3$

$C_{eff}\ V=C_1\ V+ C_2V+ C_3V$

$Ceff=C_1+C_2+ C_3$

The effective capacitance of capacitors in parallel is the sum of capacitance of individual capacitors.

27. **A capacitor of capacitor C_1 is connected to battery of potential V after charging it is disconnected from battery and connected to an uncharged capacitor of capacitanceC_2. Obtain expression for common potential, initial and final energy stored, energy lost. How do you account for energy loss? In what form energy is lost?**



$Q_1=C_1\ V$

$U_1=\frac{1}{2}C_1V^2$ --(1)

$Q_1'=C_1\ V'$

$Q_2'=C_2\ V'$

$Q_1=Q_1'+Q_2'$

$C_1\ V=(C_1+C_2)V'$

Common potential $V'=(C_1\ V)/(C_1+C_2)$ ---(2)

$U_f=1/2\ (C_1+C_2)\ (V')^2$

$=\frac{(C_1+C_2)}{2}[\frac{C_1\ V}{C_1+C_2}]^2$

$U_f=\frac{1}{2}[\frac{C_1^2\ V^2}{C_1+C_2}]$

Energy lost

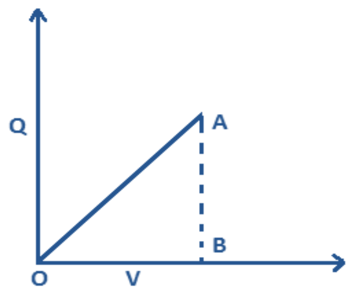
$Ui-Uf=1/2\ C_1V^2-\frac{1}{2}[\frac{C_1^2\ V^2}{C_1+C_2}]$

$=\frac{1}{2}V^2[C_1^2+C_1\ C_2-C_1^2)/(C_1+C_2)]$

$U\_lost=\frac{1}{2}\frac{C_1C_2}{C_1+C_2}V^2$

There is a transient period before the system settles to the situation stable. During this period a transient current flows from the first capacitor to the second. Energy is lost during this time in the form of heat and electromagnetic radiation.

28 Draw Q Vs V graph and find WD from the graph



Area under the graph =1/2×OB×AB

=1/2 QV

Area under graph gives WD

W=1/2 QV Where Q=CV

$W= \frac{1}{2}CV^2$

29. A capacitor of capacitance C_1 is connected to battery of potential V_1. A capacitor of capacitance C_2 is connected to battery of potential V_2, after charging they, disconnected from battery ,connected in parallel obtain exp for common potential, energy stored initially, energy stored finally, and energy lost

$V' = (C_1 V_1 + C_2 V_2)/(C_1 + C_2)$;

$U(lost) = 1/2 \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$ [To be derived]

30. A capacitor of capacitance C, a dielectric slab fully fills the gap between plates of the capacitor ε_r or K=10 . Without disconnecting battery

a) After disconnecting battery

How will the following quantities change?

i) Capacitance ; (ii) Energy stoned ;(iii) Electric field (iv)Potential diff (v)Energy density (vi)Charge stored

a) **Without disconnecting battery.**

V remains const

Ans : $Ca= (A\varepsilon_o)/d, Cm=(A\varepsilon_o\varepsilon_r)/d$

$Cm/Ca=\varepsilon r=10$

C increases by 10 times

$Ca= (A\varepsilon_o)/d, Cm=(A\varepsilon_o\varepsilon_r)/d$

$Cm/Ca=\varepsilon r=10$

C increases by 10 times

$Q_a=Ca\ V\ ; Qm=Cm\ V$

$Qa/Qm =Ca/Cm =1/10 \quad ; Qm/Qa =10$

Charge stored increases by 10 times

$E= V/d$ remain const

$Ua=1/2\ Ca\ V^2 \quad ; Um=1/2\ Cm\ V^2;$

$Um/Ua=Cm/Ca =10$

Energy stored increases by 10 times

$u= U/Volume$

$u_m/u_a =U_m/U_a =C_m/C_a =10$

Energy density increases by 10 times

b) **After disconnecting battery**

Q remains const

$Ca=(A\varepsilon_o)/d, Cm=(A\varepsilon_o\varepsilon_r)/d$

$Cm/C_a =\varepsilon_r=10$

C increases by 10 times

$$Ea=\frac{\sigma}{\varepsilon_0}\ ,Em=\frac{\sigma}{\varepsilon_0\varepsilon_r}$$

$Em/Ea =1/\varepsilon_r =1/10$

(E F decreases by 10 times

$Va=Ea\ d\ ; Vm=Em\ d$

$Vm/Va =Em/Ea =1/\varepsilon_r =1/10$

pd decreases by 10 times

$Ua=1/2\ Q^2/Ca\ ; \quad Um=1/2\ Q^2/Cm\ ; \quad Um/Ua =Ca/Cm =1/10$

energy stored decreases by 10 times.

u= U/Vol

$u_m/u_a = U_m/U_a = Ca/Cm = 1/10$

Energy density decreases by 10 times.

31. **TRY YOUR SELF:**

A capacitor has capacitance C. The dist between the 2 plates is halved, gap is fully filled with dielectric of $\varepsilon_r = 20$

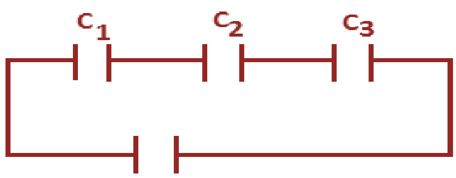
Without disconnecting battery

a) After disconnected battery. How will the following quantity change?

i. C ii. Q iii. EF iv. V v. U vi. u

32. **Energy stored in series, parallel platecapacitors**

Series:



$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

In series, Q is same

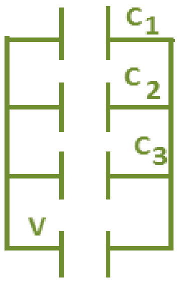
$$Us = 1/2\ Q^2/Cs = 1/2\ Q^2\ [\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}]$$

$= 1/2\ Q^2/C_1 + 1/2\ Q^2/C_2 + 1/2\ Q^2/C_3$

$Us = U_1 + U_2 + U_3$

Total energy stored in series combination is sum of energy stored in individual.

Energy stored in capacitors connected in parallel.



$C_P = C_1 + C_2 + C_3$

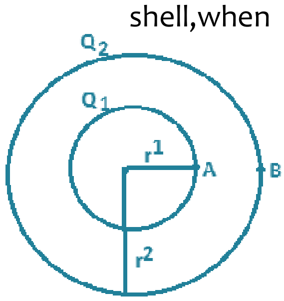
$Up = 1/2\ Cp\ V^2$

$Up = 1/2\ (C_1 + C_2 + C_3)\ V^2$

$= 1/2\ C_1\ V^2 + 1/2\ C_2\ V^2 + 1/2\ C_3\ V^2 = U_1 + U_2 + U_3$

Total energy stored in parallel combination is sum of energy stored in individual capacitors.

33. Two concentric charged spherical shells having charge Q_1, Q_2 radius r_1, r_2 are connected by wire show that charge flows from inner to outer shell,when connected by wire, irrespective of charge present on outer shell.



$V_A = (kQ_1)/r_1 + (kQ_2)/r_2$

$V_B = (kQ_1)/r_2 + (kQ_2)/r_2$

$V_A - V_B = kQ_1 \left(\frac{1}{r_1} - \frac{1}{r_2}\right) > 0$

$V_A > V_B$

Charge flows from inner to outer shell irrespective of charge stored on outer shell.