SUBJECT: MATHEMATICS (041)
SAMPLE QUESTION PAPER
CLASS- XII (2026 - 27)

Maximum Marks:80 **Time Allowed: 3 Hours**

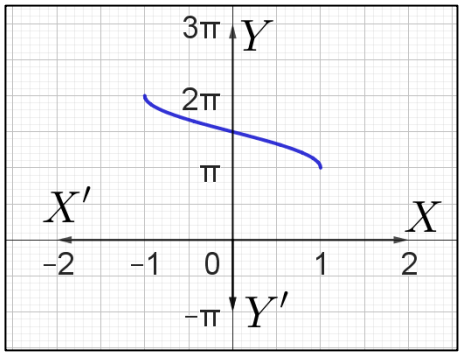
General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 38 questions. All questions are compulsory.
2. This question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Question no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Question no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Question no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Question no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Question no. 36 to 38 are case study-based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one sub-part each in 2 questions of Section E.
9. Draw neat and clean figures wherever required. Take $\pi = \frac{22}{7}$ wherever required, if not stated.
10. Use of calculators is not allowed.

SECTION – A **(20 x 1 = 20)**

This section comprises of 18 multiple choice questions and two assertion and reason type questions of 1 mark each.

Q.No.	Questions	Marks
Q1.	Identify the function whose graph is given below:  (A) $\cos^{-1}\left(\frac{x}{2}\right)$ (B) $\cos^{-1}(2x)$ (C) $\pi - \cos^{-1} x$ (D) $\pi + \cos^{-1} x$	**1**

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	For Visually Impaired: Find the range of $\frac{\pi}{2} + \sec^{-1}\left(\frac{x}{2}\right)$ (A) $[0, \pi]$ (B) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ (C) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (D) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}$	
Q2.	Domain of the function: $\sec^{-1}(2x)$ is (A) $(-\infty, \infty)$ (B) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (C) $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$ (D) $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$	**1**
Q3.	Value of $\cot^{-1}\left(2\cos\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$ (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$	**1**
Q4.	If $A = \begin{bmatrix} 2 & 1 & -2 \\ 3a & 0 & 4 \\ b & 2c & 1 \end{bmatrix}$ is a symmetric matrix, then value of $\sqrt{6a + 2b + 3c}$ is: (A) -2 (B) 2 (C) ± 2 (D) 4	**1**
Q5.	If P and Q are matrices of same order such that $P = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$, $\lvert Q \rvert = 3$, then find $\lvert adj(PQ) \rvert$ (A) -3 (B) 1 (C) 6 (D) 36	**1**
Q6.	If $\begin{bmatrix} 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix} = O$, then value of x is (A) -2 (B) -1 (C) 1 (D) 2	**1**
Q7.	The derivative of $\cos^{-1}(2x^2 - 1)$ with respect to $\cos^{-1}(x)$ is (A) 2 (B) ± 1 (C) $\frac{1}{2}$ (D) -3	**1**
Q8.	If a, b, c are arbitrary constants such that the function defined by $f(x) = \begin{cases} ax^2 + b\,,\ b \neq 0\ ,\ x \leq 1 \\ bx^2 + ax + c\,, x > 1 \end{cases}$ is differentiable at $x = 1$, then which of the following could be a triplet for (a, b, c)? (A) (3,2,0) (B) (0,0,3) (C) $\left(\frac{3}{2}, 2, 0\right)$ (D) $\left(3, \frac{3}{2}, 0\right)$	**1**
Q9.	If $y = a\left\{x + 2a\ \sin\left(\frac{x}{a}\right)\right\}, a > 0, x \in [0, \pi a]$; is a decreasing function, then the interval in which x lies is (A) $\left(\frac{5\pi}{6}, \pi\right]$ (B) $\left(\frac{2\pi}{3}, \pi\right]$ (C) $\left(\frac{5\pi a}{-6}, \pi a\right]$ (D) $\left(\frac{2\pi a}{3}, \pi a\right]$	**1**

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Q10.	The values of x for which the derivative of the function $y = x^4$ is greater than the value of the function are (A) $0 < x < 4$ (B) $0 < x \leq 4$ (C) $x < 0 \; or \; x > 4$ (D) $x < 0 \; or \; x \geq 4$	**1**
Q11.	The degree and order of the differential equation $y - x\left(\frac{dy}{dx}\right)^3 = a\left(y + \frac{d^2y}{dx^2}\right)^2$ are respectively (A) $3, 1$ (B) $2, 2$ (C) $3, 2$ (D) $4, 2$	**1**
Q12.	If $\int \frac{1}{1+e^{2x}} dx = x - P \; log(1 + e^{2x}) + C$, then '$P$' is equal to (A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$	**1**
Q13.	If vector $2\hat{\imath} - u\hat{\jmath} + 3\hat{k}$ has magnitude 4, then value of 'u' is (A) 1 (B) $\sqrt{2}$ (C) $\pm\sqrt{3}$ (D) ± 2	**1**
Q14.	Angle made by the vector $\hat{\imath} + \hat{\jmath} + \sqrt{2}\,\hat{k}$ with positive direction of z-axis is (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$	**1**
Q15.	A vector perpendicular to the vectors $\hat{\imath} + \hat{\jmath}$ and $\hat{\imath} - \hat{\jmath}$ is (A) $-2\,\hat{\jmath}$ (B) $3\hat{k}$ (C) $2\hat{\imath}$ (D) $2\hat{\imath} + \hat{k}$	**1**
Q16.	The probability that it rains on a particular day is $\frac{1}{5}$. Assuming weather conditions of any two days are independent, find the probability that it rains on three consecutive days. (A) $\frac{4}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{25}$ (D) $\frac{1}{125}$	**1**
Q17.	A die is thrown twice. What is the probability that at least one number is 4, given the sum of the numbers appearing is a multiple of 5? (A) $\frac{3}{5}$ (B) $\frac{4}{5}$ (C) $\frac{3}{7}$ (D) $\frac{4}{7}$	**1**
Q18.	A and B are two independent events such that $P(A) = \frac{1}{6}, P(B) = k$. If the probability of occurrence of exactly one of them is $\frac{1}{3}$, then find the value of k. (A) $\frac{1}{4}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{5}{6}$	**1**

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	Assertion-Reason Based Questions **Directions: Question numbers 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).** **Select the correct answer from the options (A), (B), (C) and (D) as given below:** (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A). (C) Assertion (A) is true but Reason (R) is false. (D) Assertion (A) is false but Reason (R) is true.	
Q19.	Assertion: The value of the integral $\int_{-1}^{1}\|x-2\|\,dx$ is 4 Reason: If $f(-x) = f(x)$ then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$	**1**
Q20.	Assertion: The general solution of a differential equation represents a family of curves. Reason: The derivative of an arbitrary constant is zero.	**1**
SECTION – B (5 x 2 = 10) **This section comprises of 5 very short answer (VSA) type questions of 2 marks each**		
Q21 (A).	Evaluate $\int \frac{1}{x\sqrt{x^6-1}}dx$	**2**
	OR	
Q21 (B).	Evaluate $\int \frac{\cos x-\sin x}{\sqrt{\sin(2x)}}\,dx$	
Q22.	If $y = \sin(x+y)$ and $\frac{dy}{dx} = 0$, then prove that $y = \pm 1$.	**2**
Q23 (A).	A line passing through the points $(2,6,5), and\ (3,p,7)$ is parallel to the line joining the points $(-4,3,4)$ and $(-1,0,q)$. Find the value of $\frac{q}{p}$.	**2**
	OR	
Q23 (B).	A rectangle ABCD is such that $\overrightarrow{AC} = \vec{u}, and\ \overrightarrow{BD} = \vec{v}$, then find the vectors representing four sides of the rectangle (in a cyclic order starting from $\overrightarrow{AB}$).	
Q24.	For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\|\vec{a}\| = \sqrt{6}$, $\vec{a}\cdot\vec{b} = 1$ and $\vec{b}\times\vec{a} = 3\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$. Find $\|\vec{b}\|$.	**2**

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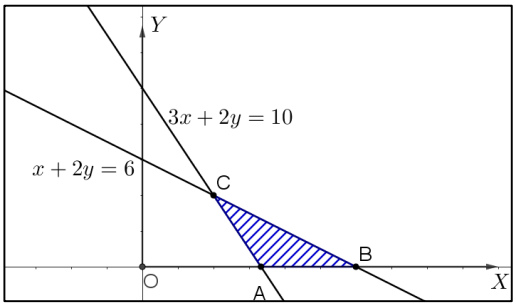
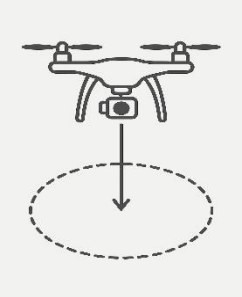
Q25.	The feasible region of a linear programming problem is shown in the given figure:  Write all the constraints on the decision variables involved **For Visually Impaired:** For a linear programming problem, the maximum value of an objective function $Z = 3x + 4y$ exists at the points (p, q) and $\left(2p, \frac{q}{2}\right)$. Find the relation between p and q.	**2**
	SECTION – C (6 x 3 = 18) **This section comprises of 6 short answer (SA) type questions of 3 marks each.**	
Q26.	For two matrices, $P = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}$, find PQ and QP, whichever is defined. Also, find the inverse of the product matrix, if it exists.	**3**
Q27.	The circular area of vision on the ground by a surveillance drone fitted with camera increases at the rate of $44\ m^2$ per metre as it gains vertical height. The radius of the circle on the ground increases at the rate of 6 m/s. Find the rate at which the height of the drone increases with time, at the instant when the radius of the circular area on the ground is 14 m. $\left(Use\ \pi = \frac{22}{7}\right)$ 	**3**
Q28 (A).	If $x = sin^3 t, y = cos^3 t$, $Evaluate\ \frac{d^2y}{dx^2}\ at\ \left(\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$.	**3**
	OR	
Q28 (B).	If $y = x^{logx} + (logx)^x$; $x > 1$, then evaluate $\frac{dy}{dx}$	
Q29 (A).	Find the particular solution of the differential equation $xdy + (y - x^2e^x)dx = 0$, given that y (1) =0	**3**
	OR	
Q29 (B).	Find the particular solution of the differential equation $(x^2 + y^2)dy = xydx$, given that $y(0) = 1$	

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Q30 (A).	Find the foot of the perpendicular drawn from the point (2,1,4) on the line $x-2=\frac{2y+3}{4}=2-z$. Also find the equation of the line passing through this point and the point (2,1,4).	**3**
	OR	
Q30 (B).	Find the angle between the lines: $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$ and $\frac{x-5}{2}=y-\frac{1}{2}=\frac{3-z}{2}$. Also find the cartesian equation of a line passing through the point (1,1,1) and perpendicular to both lines.	
Q31.	Solve the linear programming problem graphically: Maximize $Z=40x+30y$ Subject to constraints: $x+y\leq 6, 2x\geq y, y\geq 2$ **For Visually Impaired:** For a linear programming problem, vertices of the feasible region are $(20,0),(40,0),(30,10),(0,20)$ and $(0,10)$. Minimize and maximize the objective function $Z=50x-20y$	**3**
	SECTION – D (4 x 5 = 20) **This section comprises of 4 long answer (LA) type questions of 5 marks each**.	
Q32 (A).	Let $A=N\times N$, where N is the set of natural numbers and a relation R is defined on A such that $(a,b)R(c,d)\ if\ and\ only\ if\ a^2+d^2=b^2+c^2$. Check whether R is an equivalence relation or not?	**5**
	OR	
Q32 (B).	A relation $R:(-\infty,\infty)\to[0,\infty)$ is defined as $R=\{(x,y):y=x^2+5\}$ Draw its graph and check whether it is a function or not. If the given relation is a function, then examine whether the function is injective, and whether it is surjective or not. **For Visually Impaired:** A relation $R:(-\infty,\infty)\to[0,\infty)$ is defined as $R=\{(x,y):y=x^2+7\}$. Check whether it is a function or not. If the given relation is a function, then examine whether the function is injective, surjective, or both, and conclude whether it is bijective or not.	
Q33.	Evaluate $\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx$	**5**
Q34 (A).	Using Integration, find the area of the region $\{(x,y):0\leq y\leq\sqrt{4-x^2}, x\geq 2-y\}$.	**5**
	OR	
Q34 (B).	Using Integration, find the area of the region bounded by the curve $y^2=2x$ and the line $x-2y=0$.	

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	For Visually Impaired:	
Q34 (A).	Using Integration, find the area of the region enclosed within $25x^2 + 16y^2 = 400$	**5**
	OR	
Q34 (B).	Using Integration find the area of the region in the first quadrant bounded by the curve $y^2 = 8x$, and the lines $x = 2, x = 5$ and x-axis.	
Q35.	A fitness app syncs with four types of smartwatches: Alpha, Beta, Gamma and Delta. 45% of the users use Alpha, 30% use Beta,15% use Gamma and 10% use Delta. Due to device errors, the probabilities of getting a false heart-rate alert from the Alpha, Beta, Gamma and Delta smartwatches are respectively 0.01, 0.02, 0.05, and 0.01. Based on the given information, answer the following questions: (i) Find the probability that a randomly selected user gets a false alert. (ii) If a false alert occurs, what is the probability that it came from a Beta smartwatch?	**5**

SECTION – E (3 x 4 = 12)

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each.

Q36. Two security sensors in a multi-storey mall emit laser beams for movement detection. Three of these beams travel along the following lines:

$l_1: \vec{r} = (\lambda + 1)\hat{\imath} + (3\lambda + 2)\hat{\jmath} + (3 - \lambda)\hat{k}$ and

$l_2: \vec{r} = 2(\mu + 1)\hat{\imath} + (3\mu - 1)\hat{\jmath} + (\mu + 1)\,\hat{k}$

$l_3: \vec{r} = (\gamma + 3)\hat{\imath} + (3\gamma - 1)\hat{\jmath} + (1 - \gamma)\,\hat{k}$



Based on the given information, answer the following questions:

(i) Show that the lines l_1 and l_2 are not parallel to each other. **1**

(ii) Show that the lines l_1 and l_3 are parallel to each other. **1**

(iii) (a) Find the shortest distance between the lines l_1 and l_2.

OR

(b) Find the shortest distance between the lines l_1 and l_3. **2**

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Q37.	Water is essential for life, and rivers are vital water sources across the world. The Yamuna Action Plan (YAP), one of India's largest river restoration projects, was launched to clean the Yamuna, but its effectiveness (WQI) the Water Quality Index must be measured through cost-benefit, economic and environmental analysis. Therefore, comprehensive pollutant treatment, prevention of further contamination, and community awareness are crucial for sustainable river management. Implementing the same concept to the river Yamuna, if an initial base line WQI is 35, a quadratic expression is formed to measure the impact of investment of ₹x crore on the WQI i.e., $W(x)$ as $W(x) = 35 + 1.6x - 0.02x^2$, where x is in Rupees in crores  Use the given $W(x)$ to answer the following questions: (i) Find the value of x at the stationary point for the function $W(x)$. (ii) Find the maximum value of the WQI using the second derivative test? (iii) (a) If the investment is ₹ 35 crores, find by how much the WQI is short of its maximum achievable value. **Or** (b) Evaluate the WQI when the investment is ₹60 crore. Determine whether this indicates an enhancement or a diminishing return in our objective when compared with the optimum WQI.	**1** **1** **2**
Q38.	To celebrate their child's birthday, a family decided to distribute fruits among children in three orphanages. The fruit vendor packed three different types of baskets, according to the quantity required at each orphanage, details are given as follows: Basket A contains 5 kg Apples, 10 kg Oranges and 5 kg Pomegranate Basket B contains 10 kg Apples, 10 kg Oranges and 5kg Pomegranate Basket C contains 8 kg Apples, 15 kg Oranges and 7kg Pomegranate Cost of basket A, B and C are respectively ₹2750, ₹3500, and ₹4100.  Based on the given information, answer the following questions: (i) Write the matrix form of the equations involved in this situation and find the adjoint of the coefficient matrix involved. (ii) Using inverse of the matrix, calculate the price of each fruit per kg.	**2** **2**

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MATHEMATICS (041)
MARKING SCHEME
CLASS XII (2026-27)

SECTION: A (Solution of MCQs of 1 Mark each, no part marks)

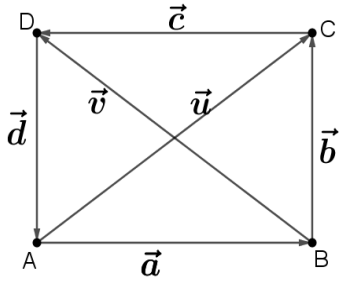
Q. No.	ANS	HINTS/SOLUTION	Marks
1.	**(D)**	$\pi + \cos^{-1} x$	**1**
	(D)	**For Visually Impaired:** $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}$	**1**
2.	**(D)**	$\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$	**1**
3.	**(A)**	$\cot^{-1}\left(2\cos\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right) = \cot^{-1}\left(2\cos\left(\frac{1}{2}\times\frac{\pi}{3}\right)\right)$ $= \cot^{-1}\left(2\cos\left(\frac{\pi}{6}\right)\right) = \cot^{-1}\left(2\times\frac{\sqrt{3}}{2}\right) = \cot^{-1}(\sqrt{3}) = \frac{\pi}{6}$	**1**
4.	**(B)**	$3a = 1, b = -2, 2c = 4 \Rightarrow \sqrt{6a + 2b + 3c} = \sqrt{2 - 4 + 6} = \sqrt{4} = 2$	**1**
5.	**(C)**	$\lvert adj(PQ)\rvert = \lvert PQ\rvert = \lvert P\rvert\lvert Q\rvert = 2 \times 3 = 6$ ($\because$ order of P and $Q = 2 \times 2$)	**1**
6.	**(D)**	$\begin{bmatrix}1 & x\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}4\\ -1\end{bmatrix} = O \Rightarrow \begin{bmatrix}1 & 2x\end{bmatrix}\begin{bmatrix}4\\ -1\end{bmatrix} = O \Rightarrow 4 - 2x = 0 \Rightarrow x = 2$	**1**
7.	**(A)**	Let $y = \cos^{-1}(2x^2 - 1)$ and $u = \cos^{-1}(x)$ $\Rightarrow y = \cos^{-1}(2\cos^2 u - 1)$ as $\cos u = x$ $\Rightarrow y = \cos^{-1}(\cos(2u)) = 2u$ $\Rightarrow \frac{dy}{du} = 2$	**1**
8.	**(D)**	$f(x)$ is differentiable $\Rightarrow f(x)$ is continuous $\Rightarrow c = 0$ and $a = 2b$ $\Rightarrow (a, b, c)$ becomes $(2b, b, 0)$ which is satisfied by option (D) $(3, \frac{3}{2}, 0)$ only	**1**
9.	**(D)**	y is decreasing $\Rightarrow \frac{dy}{dx} < 0 \Rightarrow a\{1 + 2\cos\left(\frac{x}{a}\right)\} < 0 \Rightarrow \cos\left(\frac{x}{a}\right) < -\frac{1}{2}$ $\Rightarrow \frac{x}{a} \in \left(\frac{2\pi}{3}, \pi\right] \Rightarrow x \in \left(\frac{2\pi a}{3}, \pi a\right]$	**1**
10.	**(A)**	$\frac{dy}{dx} > y \Rightarrow 4x^3 > x^4 \Rightarrow x^3(4 - x) > 0 \Rightarrow x(x - 4) < 0 \Rightarrow 0 < x < 4$	**1**
11.	**(B)**	The order is 2 and the term containing $\frac{d^2y}{dx^2}$ has the maximum power 2 in the equation, so the degree is 2	**1**
12.	**(D)**	$\int \frac{1}{1+e^{2x}}dx = \int\left(\frac{1+e^{2x}}{1+e^{2x}} - \frac{e^{2x}}{1+e^{2x}}\right)dx = x - \frac{1}{2}log\lvert 1 + e^{2x}\rvert + C$ So $P = \frac{1}{2}$	**1**
13.	**(C)**	Magnitude of $2\hat{\imath} - u\hat{\jmath} + 3\hat{k}$ = 4 $\Rightarrow \sqrt{4 + u^2 + 9} = 4$ $\Rightarrow u^2 = 16 - 13 = 3 \Rightarrow u = \pm\sqrt{3}$	**1**
14.	**(B)**	Direction cosines of the vector $\hat{\imath} + \hat{\jmath} + \sqrt{2}\,\hat{k}$ are $\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$ Let angle = γ. Then $\cos\gamma = \frac{1}{\sqrt{2}} \Rightarrow \gamma = \frac{\pi}{4}$	**1**
15.	**(B)**	Both the vectors $\hat{\imath} + \hat{\jmath}$ and $\hat{\imath} - \hat{\jmath}$ are in XY-plane So a vector perpendicular to both is $3\hat{k}$	**1**

16.	**(D)**	The probability that it rains on a particular day is $\frac{1}{5}$. Probability that it rains on three consecutive days = $\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}=\frac{1}{125}$	**1**
17.	**(D)**	Let the events be A: sum of the numbers appeared is a multiple of 5 B: at least one number is 4 $A=\{(1,4),(2,3),(3,2),(4,1),(4,6),(5,5),(6,4)\}$ $A\cap B=\{(1,4),(4,1),(4,6),(6,4)\}$ So required probability = $P(B\|A)=\frac{4}{7}$	**1**
18.	**(A)**	Probability of occurrence of any one of them is $\frac{1}{3}$ $\Rightarrow P(A'\cap B)+P(A\cap B')=\frac{1}{3}\Rightarrow P(A')P(B)+P(A)P(B')=\frac{1}{3}$ $\Rightarrow\left(1-\frac{1}{6}\right)k+\frac{1}{6}(1-k)=\frac{1}{3}\Rightarrow k\left(\frac{5}{6}-\frac{1}{6}\right)=\frac{1}{3}-\frac{1}{6}\Rightarrow k=\frac{1}{6}\times\frac{6}{4}=\frac{1}{4}$	**1**
19.	**(B)**	Both assertion and reason are true, but reason is not the correct explanation of the assertion $\int_{-1}^{1}\|x-2\|\,dx=\int_{-1}^{1}(-x+2)\,dx=4$	**1**
20.	**(A)**	Both assertion and reason are true, and reason is the correct explanation of the assertion.	**1**

Section –B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21(A).	$I=\int\frac{1}{x\sqrt{x^6-1}}dx=\frac{1}{6}\int\frac{6x^5}{x^6\sqrt{x^6-1}}dx$ Let $x^6-1=t^2 \quad\Rightarrow 6x^5dx=2tdt$ $I=\frac{1}{3}\int\frac{1}{(t^2+1)}dt=\frac{1}{3}\tan^{-1}\left(\sqrt{x^6-1}\right)+C$	½ 1+½
	OR	
21(B).	Let $I=\int\frac{\cos x-\sin x}{\sqrt{\sin(2x)}}\,dx=\int\frac{\cos x-\sin x}{\sqrt{1+\sin(2x)-1}}\,dx=\int\frac{\cos x-\sin x}{\sqrt{(sinx+cosx)^2-1}}\,dx$ Let $sinx+cosx=t \quad\Rightarrow(\cos x-\sin x)dx=dt$ $I=\int\frac{1}{\sqrt{t^2-1}}\,dt=log\left\|t+\sqrt{t^2-1}\right\|+C$ $=log\left\|sinx+cosx+\sqrt{(sinx+cosx)^2-1}\right\|+C$ $=log\left\|sinx+cosx+\sqrt{\sin(2x)}\right\|+C$	½ ½ ½ ½
22.	$y=\sin(x+y)\Rightarrow\sin^{-1}y=x+y$ $\Rightarrow\frac{1}{\sqrt{1-y^2}}=\frac{dx}{dy}+1\ \Rightarrow\ \frac{dx}{dy}=\frac{1-\sqrt{1-y^2}}{\sqrt{1-y^2}}$ $\Rightarrow\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{1-\sqrt{1-y^2}}$ So, $\frac{dy}{dx}=0\ \Rightarrow y=\pm1$	**1** **1**

	Alternative way: $y = \sin(x+y) \Rightarrow \frac{dy}{dx} = \cos(x+y)\left(1+\frac{dy}{dx}\right)$ $\Rightarrow \frac{dy}{dx}(1-\cos(x+y)) = \cos(x+y)$ $\Rightarrow \cos(x+y) = 0$ as $\frac{dy}{dx} = 0$ $\Rightarrow \sin(x+y) = \pm 1 \Rightarrow y = \pm 1$	**1** **1**
23(A).	Given points are $A(2,6,5), B(3,p,7), C(-4,3,4)$ and $D(-1,0,q)$ Direction ratios of the line $\overleftrightarrow{AB}$ are $\langle 1,\ p-6,\ 2\rangle$ and Direction ratios of the line $\overleftrightarrow{CD}$ are $\langle 3,-3,q-4\rangle$ As the lines are parallel, so we have $\frac{1}{3} = \frac{p-6}{-3} = \frac{2}{q-4}$ $\Rightarrow -1 = p-6 \ and\ q-4 = 3\times 2$ $\Rightarrow p = -1+6 = 5,\ q = 6+4 = 10$ Thus, $\frac{q}{p} = \frac{10}{5} = 2$	**1** **½** **½**
	OR	
23 (B).	 Let vectors representing the four sides of the rectangle are as follows $\overrightarrow{AB} = \vec{a}, \overrightarrow{BC} = \vec{b}, \overrightarrow{CD} = \vec{c}, and\ \overrightarrow{DA} = \vec{d}$ Then $\overrightarrow{AC} = \vec{u} = \vec{a}+\vec{b}$ …(i)and $\overrightarrow{BD} = \vec{v} = \vec{b}+\vec{c} = \vec{b}-\vec{a}$ …(ii) Solving (i) and (ii), $\vec{a} = \frac{1}{2}(\vec{u}-\vec{v})\ and\ \vec{b} = \frac{1}{2}(\vec{u}+\vec{v})$ Also, $\vec{c} = -\vec{a} = \frac{1}{2}(\vec{v}-\vec{u})$ and $\vec{d} = -\vec{b} = -\frac{1}{2}(\vec{u}+\vec{v})$	**½** **1** **½**
24.	Given $\lvert\vec{a}\rvert = \sqrt{6}$, $\vec{a}\cdot\vec{b} = 1$ and $\vec{b}\times\vec{a} = 3\hat{\imath}+4\hat{\jmath}-2\hat{k}$ So $\lvert\vec{a}\times\vec{b}\rvert = \lvert\vec{b}\times\vec{a}\rvert = \sqrt{29}$ We know, $\lvert\vec{a}\times\vec{b}\rvert^2 + \lvert\vec{a}\cdot\vec{b}\rvert^2 = \lvert\vec{a}\rvert^2\lvert\vec{b}\rvert^2$ $\Rightarrow 29+1 = 6\lvert\vec{b}\rvert^2 \Rightarrow \lvert\vec{b}\rvert^2 = \frac{30}{6} = 5 \Rightarrow \lvert\vec{b}\rvert = \sqrt{5}$	**½** **½** **1**
25.	Constraints are $3x+2y \geq 10, x+2y \leq 6,$ $y \geq 0$	**1½** **½**

	For Visually Impaired: As maximum value exists at (p, q) and $\left(2p, \frac{q}{2}\right)$ So $Z\ at\ (p,q) = Z\ at\left(2p, \frac{q}{2}\right)$ $\Rightarrow 3p + 4q = 3 \times 2p + 4 \times \frac{q}{2}$ $\Rightarrow 3p + 4q = 6p + 2q$ $\Rightarrow 6p - 3p = 4q - 2q$ $\Rightarrow 3p = 2q$	**1** **1**

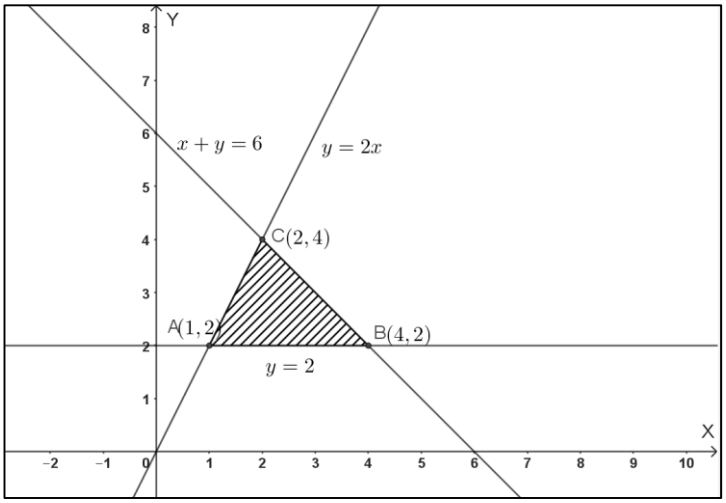
Section –C

[This section comprises of solution of short answer type questions (SA) of 3 marks each]

26(A)	Let $A = PQ$ and $B = QP$. Then $A = PQ = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$ $B = QP = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 4 \\ 2 & 0 & 0 \end{bmatrix}$ Here $\lvert A\rvert = 4 \neq 0$ So, A is a non-singular matrix, hence its inverse exists. Now $A^{-1} = \frac{1}{\lvert A\rvert} adj\ A = \frac{1}{4}\begin{bmatrix} 2 & 0 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ Here $\lvert B\rvert = 0$ So, B is a singular matrix, hence its inverse does not exist.	**1** **1** **½** **½**
27.	Let A be the area of the circular region covered by the drone and let r be its radius. Let h be the height of the drone at any time t. Given $\frac{dA}{dh} = 44\ m^2\ per\ metre$ and $\frac{dr}{dt} = 6\ m/s$ To find $\frac{dh}{dt}$ when r = 14 m. $A = \pi r^2 \quad \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \quad \Rightarrow \frac{dA}{dh}.\frac{dh}{dt} = 2\pi r \frac{dr}{dt}$ $44\ \frac{dh}{dt} = 2(\frac{22}{7})6r \quad \Rightarrow \left.\frac{dh}{dt}\right]_{r=14m} = 12$ m/s The height of the drone increases at the rate of 12 m/s at that moment.	**½** **1 ½** **1**
28(A).	$\frac{dx}{dt} = 3sin^2t\ cos\ t$ and $\frac{dy}{dt} = -3\ cos^2t\ sin\ t$ $\frac{dy}{dx} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right. = -cot\ t$ $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right).\frac{dt}{dx} = cosec^2t\left(\frac{1}{3sin^2t\cos t}\right) = \frac{1}{3}(sec\ t.cosec^4t)$ At the point $\left(\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$, $sin^3t = \frac{1}{2\sqrt{2}} \Rightarrow t = \frac{\pi}{4}$ $\left.\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}} = \frac{4\sqrt{2}}{3}$	**1** **1** **1**

	OR	
28(B).	Let $y = x^{logx} + (logx)^x = u + v$ where $u = x^{logx}$ and $v = (logx)^x$ $log\ u = (log\ x)^2$ and $log\ v = x.log(log\ x)$ $\frac{1}{u}\frac{du}{dx} = 2(\ log\ x).\frac{1}{x}$ and $\frac{1}{v}\frac{dv}{dx} = log(log\ x) + \frac{1}{log\ x}$ $\frac{du}{dx} = x^{logx}\left(2(\ log\ x).\frac{1}{x}\right)$ and $\frac{dv}{dx} = (logx)^x\left(log(log\ x) + \frac{1}{log\ x}\right)$ $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $\Rightarrow \frac{dy}{dx} = \frac{2}{x}\ x^{logx}\ log\ x + \ (logx)^x\left(log(log\ x) + \frac{1}{log\ x}\right)$ **Alternative Solution:** Since $x = e^{logx}$ $y = x^{logx} + (logx)^x = e^{\log(x^{logx})} + e^{\log((logx)^x)}$ $= e^{(\log x)^2} + e^{x\log(\log x)}$ $\frac{dy}{dx} = e^{(\log x)^2}\left[\frac{2\log x}{x}\right] + e^{x\log(\log x)}[\log(\log x) + x\left(\frac{1}{\log x}\right)\left(\frac{1}{x}\right)]$ $OR\ \ x^{logx}\left[\frac{2\log x}{x}\right] + (logx)^x[\log(\log x) + \left(\frac{1}{\log x}\right)]$	**1** **1** **1** **1** **1+1**
29(A).	$xdy + (y - x^2e^x)dx = 0$ is a linear differential equation given by $\frac{dy}{dx} + \frac{1}{x}\ y = xe^x$ I.F = $e^{\int\frac{1}{x}dx} = \ e^{log\ x} = x$ Solution is $y.x = \int x^2e^x\ dx$ using by parts $yx = x^2e^x - \int 2xe^x\ dx$ using by parts again $yx = x^2e^x - 2\{xe^x - e^x\} + C$ y (1) = 0 $\Rightarrow\ \ C = -e$ Particular solution is $yx = x^2e^x - 2\{xe^x - e^x\} - e = e^x\{x^2 - 2x + 2\} - e$	**½** **½** **½** **½** **½** **½**
	OR	
29(B).	$\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ is a homogeneous differential equation. Let $y = v\ x \qquad \Rightarrow \frac{dy}{dx} = v + x\ \frac{dv}{dx}$ So, equation reduces to	

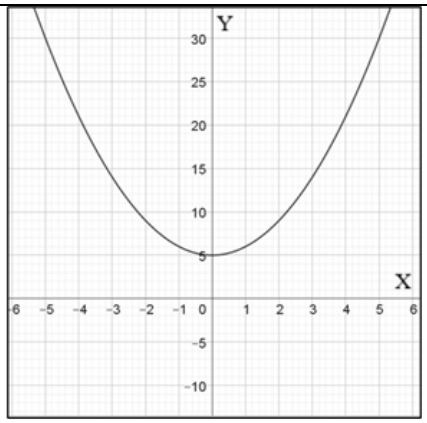
	$v + x\,\frac{dv}{dx} = \frac{v}{1+v^2} \quad \Rightarrow x\,\frac{dv}{dx} = \frac{v-v-v^3}{1+v^2}$	**1**
	$\int \frac{1+v^2}{v^3}dv = -\int \frac{dx}{x}$	
	$\int(\frac{1}{v^3}+\frac{1}{v})\ dv = -log\ \lvert x\rvert + \log C \Rightarrow \frac{-1}{2v^2} + \log\lvert v\rvert = -\log\lvert x\rvert + \log C$	
	$\log\left\lvert\frac{y}{C}\right\rvert = \frac{x^2}{2y^2}$ or $y = C\,e^{\frac{x^2}{2y^2}}$	**1**
	$C = 1$ when $x = 0$ and $y = 1$.	
	The particular solution is $y = e^{\frac{x^2}{2y^2}}$	**1**
30(A).	A point on the line $x - 2 = \frac{2y+3}{4} = 2 - z = k$ *is* $P\left(k+2, \frac{4k-3}{2}, 2-k\right)$	**½**
	And direction ratios of the given line are $\langle 1, 2, -1\rangle$.	
	Direction Ratios of the line joining the points $A(2,1,4)$ and this point P are	
	$\langle k+2-2, \frac{4k-3}{2} - 1, 2-k-4\rangle = \langle k, \frac{4k-5}{2}, -k-2\rangle$	**½**
	As $\overrightarrow{AP}$ is perpendicular to the given line, So	
	$1\times k + 2\times\frac{4k-5}{2} + (-1)\times(-k-2) = 0$	
	$\Rightarrow k + 4k - 5 + k + 2 = 0 \Rightarrow 6k - 3 = 0 \Rightarrow k = \frac{1}{2}$	**½**
	So, foot of the perpendicular is	
	$P\left(k+2, \frac{4k-3}{2}, 2-k\right) = \left(\frac{1}{2}+2, \frac{2-3}{2}, 2-\frac{1}{2}\right) = \left(\frac{5}{2}, \frac{-1}{2}, \frac{3}{2}\right)$	**½**
	Direction ratios of the line $\overleftarrow{AP}$ are $\langle k, \frac{4k-5}{2}, -k-2\rangle = \langle\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2}\rangle$ or $\langle 1, -3, -5\rangle$	**½**
	And equation of $\overleftarrow{AP}$ is	
	$\frac{x-2}{1} = \frac{y-1}{-3} = \frac{z-4}{-5}$ OR $x - 2 = \frac{1-y}{3} = \frac{4-z}{5}$	**½**
	OR	
30(B).	Given lines are $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$ and	
	$\frac{x-5}{2} = y - \frac{1}{2} = \frac{3-z}{2} \Rightarrow \frac{x-5}{2} = \frac{y-\frac{1}{2}}{1} = \frac{z-3}{-2}$	**½**
	Let the angle between the lines be θ, then	
	$\cos\theta = \frac{2\times2+2\times1+1\times(-2)}{\sqrt{9}\times\sqrt{9}} = \frac{4}{9} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{9}\right)$	**1**
	Now required line will be in the direction of the vector	
	$\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 2 & 1\\ 2 & 1 & -2\end{vmatrix} = -5\hat{\imath} + 6\hat{\jmath} - 2\hat{k}$	**1**
	So required line is $\frac{x-1}{-5} = \frac{y-1}{6} = \frac{z-1}{-2}$ OR $\frac{x-1}{5} = \frac{y-1}{-6} = \frac{z-1}{2}$	**½**

31.	 Feasible region is bounded with three vertices $A(1,2), B(4,2), C(2,4)$ Now $Z_A = 100, Z_B = 220, Z_C = 200$ So, Maximum Value of Z is 220 at the point $B(4,2)$.	**1 ½** **1** **½**
	For Visually Impaired: The vertices of the feasible region are $A(20,0), B(40,0), C(30,10),\ D(0,20)$ and $E(0,10)$. Now $Z_A = 50 \times 20 - 20 \times 0 = 1000$ $Z_B = 50 \times 40 - 20 \times 0 = 2000$ $Z_C = 50 \times 30 - 20 \times 10 = 1300$ $Z_D = 50 \times 0 - 20 \times 20 = -400$ $Z_E = 50 \times 0 - 20 \times 10 = -200$ Minimum value of Z is -400 at $D(0,20)$ Maximum Value of Z is 2000 at $B(40,0)$	**2** **1**

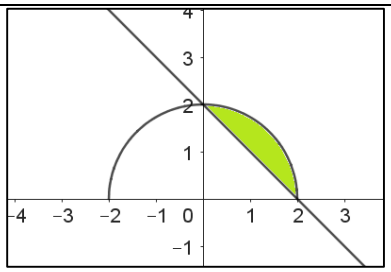
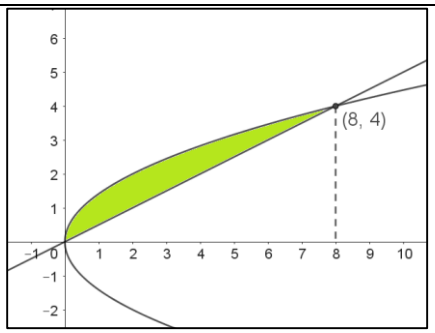
Section –D

[This section comprises of solution of long answer type questions (LA) of 5 marks each]

32(A).	Given relation R is defined on $A = N \times N$ such that $(a,b)R(c,d)\ if\ and\ only\ if\ a^2 + d^2 = b^2 + c^2$ Reflexive: Let $(p,q) \in A = N \times N$ Then as $p, q \in N$, so $p^2 + q^2 = q^2 + p^2$ $\Rightarrow (p,q)R(p,q)\ \forall (p,q) \in N \times N$ $\Rightarrow R$ is a reflexive relation. Symmetric: Let $(p,q)\ and\ (r,s) \in A = N \times N$ such that $(p,q)\ R\ (r,s)$ $\Rightarrow p^2 + s^2 = q^2 + r^2$ $\Rightarrow q^2 + r^2 = p^2 + s^2$ $\Rightarrow r^2 + q^2 = s^2 + p^2$ $\Rightarrow (r,s)\ R\ (p,q)$ Thus, R is a symmetric relation. Transitive:	**1 ½** **1 ½**

	Let $(p,q),(r,s),(u,v) \in A$ such that $(p,q)\ R\ (r,s)\ and\ (r,s)R(u,v)$ $\Rightarrow p^2+s^2=q^2+r^2$ ...(i) and $r^2+v^2=s^2+u^2$ ...(ii) Adding (i) and (ii), $p^2+s^2+r^2+v^2=q^2+r^2+s^2+u^2$ $\Rightarrow p^2+v^2=q^2+u^2$ $\Rightarrow (p,q)\ R\ (u,v)$ Thus, R is a transitive relation So, R is a reflexive, symmetric and transitive relation. Hence R is an equivalence relation	**1 ½** **½**
	OR	
32(B).	 Yes, R is a function as the graph shows every element of the set $(-\infty,\infty)$ has a unique image in the set $[0,\infty)$. Now, let us denote this function by $f:(-\infty,\infty)\rightarrow[0,\infty)$ as $f(x)=x^2+5$ Injective Function: Here $f(-1)=(-1)^2+5=6$ and $f(1)=1^2+5=6$ So $f(-1)=f(1)=6$ Thus f is a many-one function, so not an injective function. Surjective Function: Range of $f(x)=\ [5,\infty) \neq$ co-domain$=[0,\infty)$ So, it is not surjective. **Alternate Method:** Correct graph as above Yes, R is a function as the graph shows every element of the set $(-\infty,\infty)$ has a unique image in the set $[0,\infty)$. Now, let us denote this function by $f:(-\infty,\infty)\rightarrow[0,\infty)$ as $f(x)=x^2+5$ Injective Function: Let $a,b \in (-\infty,\infty)$, such that $f(a)=f(b)$ $\Rightarrow a^2+5=b^2+5 \Rightarrow a^2=b^2 \Rightarrow a=\pm b$ So f is not injective. Surjective Function: For $y=1$ there is no pre-image as if $1=x^2+5 \Rightarrow x^2=-4$ not possible. So, it is not surjective. **For Visually Impaired:** For every element a of the set $(-\infty,\infty)$, $a^2 \in [0,\infty)$ and therefore $a^2+7 \in [7,\infty)$ Hence every element of the set $(-\infty,\infty)$ has a unique image in the set $[0,\infty)$.	**1 for correct graph** **1** **1 ½** **1 ½** **1** **1** **1 ½** **1 ½**

	So, $f:(-\infty,\infty)\to[0,\infty)$, $f(x)=x^2+7$ is a function. Now, let us denote this function by $f:(-\infty,\infty)\to[0,\infty)$ as $f(x)=x^2+7$ Injective Function: Here $f(-1)=(-1)^2+7=8$ and $f(1)=1^2+7=8$ So $f(-1)=f(1)=8$ Thus f is a many-one function, so not an injective function. Surjective Function: Range of $f(x)=\ [7,\infty)\neq$ co-domain$=[0,\infty)$ So, it is not surjective. Thus $f(x)$ is neither injective nor surjective and hence not bijective	**1** **1 ½** **1 ½** **1**
33.	Let $I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx$ $I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}\times\frac{a-x}{a-x}}\,dx$ $=\int_{-a}^{a}\frac{a-x}{\sqrt{a^2-x^2}}dx=\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}}dx-\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx=I_1-I_2$ $I_1=a\int_{-a}^{a}\frac{1}{\sqrt{a^2-x^2}}dx$ $=2a\int_{0}^{a}\frac{1}{\sqrt{a^2-x^2}}dx$ using $\int_{-a}^{a}f(x)\,dx=2\int_{0}^{a}f(x)\,dx$ if $f(-x)=f(x)$ $I_1=2a\ \left[\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a=2a\left(\frac{\pi}{2}\right)=\pi a$ and $I_2=\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx=0$ using $\int_{-a}^{a}f(x)\,dx=0$ if $f(-x)=-f(x)$ $I=\ I_1-I_2=\pi a$ **Alternate Method:** Let $x=a\cos 2\theta\ \Rightarrow dx=-2\,a\sin(2\,\theta)\,d\theta$ When $x=-a,\theta=\frac{\pi}{2}$ and $x=a,\theta=0$ $I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx\ =-2\,a\int_{\frac{\pi}{2}}^{0}\tan\theta\sin(2\theta)\,d\theta$ $I=\ 2\,a\int_{0}^{\frac{\pi}{2}}\tan\theta\sin(2\theta)\,d\theta$ .....(i) using $\int_a^b f(x)dx=-\int_b^a f(x)dx$ $I=\ 2a\int_{0}^{\frac{\pi}{2}}\tan\left(\frac{\pi}{2}-\theta\right)\sin(\pi-2\,\theta)d\theta$ using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ $I=\ 2a\int_{0}^{\frac{\pi}{2}}\cot(\theta)\sin(2\theta)\,d\theta$ .....(ii) Adding equations (i) and (ii) $2I=2a\int_{0}^{\frac{\pi}{2}}(\tan\theta+\cot\theta)\sin(2\theta)\,d\theta$ $\Rightarrow I=\ 2a\int_{0}^{\frac{\pi}{2}}(sin^2\theta+cos^2\theta)\,d\theta$ $\Rightarrow I=2a\int_{0}^{\frac{\pi}{2}}1\,d\theta=2a\left(\frac{\pi}{2}\right)=\pi a$	 **½** **1** **1** **1** **½** **½** **½** **1** **1** **1** **1**

34(A).	 1 mark for correct figure $y = \sqrt{4 - x^2}$ is a semi-circle with radius 2 units and centre (0,0) $x = 2 - y$ which is x + y = 2 is a line with intercepts 2 on each axis. Curves meet at (0,2) and (2,0) Area = $\int_0^2 \left(\sqrt{4 - x^2} - (2 - x)\right) dx$ $= \left[\frac{x}{2}\sqrt{4 - x^2} + 2 \sin^{-1}\left(\frac{x}{2}\right) - 2x + \frac{x^2}{2}\right]_0^2$ $= \pi - 4 + 2 = (\pi - 2)$ sq. units	**1** **1** **1** **1** **1**
	OR	
34(B).	 1 mark for correct figure. $y^2 = 2x$ represents a parabola, whose points of intersection with the given line $x = 2y$ are $(0,0)$, $(8,4)$ Area = $\int_0^8 \left(\sqrt{2x} - \frac{x}{2}\right) dx$ $= \left[\sqrt{2}\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{4}\right]_0^8$ $= \frac{64}{3} - 16 = \frac{16}{3}$ sq. units	**1** **1** **1** **1** **1**
34(A).	**For Visually Impaired:** $25x^2 + 16y^2 = 400$ is an Ellipse with the standard equation as $\frac{x^2}{16} + \frac{y^2}{25} = 1$ $\Rightarrow y = \frac{5}{4}\sqrt{4^2 - x^2}$ Area = $4\int_0^4 \left(\frac{5}{4}\sqrt{4^2 - x^2}\right) dx$ $= 5\int_0^4 \left(\sqrt{4^2 - x^2}\right) dx$ $= 5\left[\frac{x}{2}\sqrt{16 - x^2} + 8 \sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$ $= 5[4(\pi)] = 20\pi$ sq. units	**½** **1** **1** **1½** **1**
	OR	

34(B).	$y^2 = 8x$ represents a parabola, and the area of the region above the x-axis, below the parabola, between the lines $x = 2$ and $x = 5$ is given by Area = $\int_2^5 (\sqrt{8x})\ dx$ $= \left[2\sqrt{2}\frac{2}{3}x^{\frac{3}{2}}\right]_2^5$ $= \sqrt{2}\left(\frac{4}{3}\right)[5\sqrt{5} - 2\sqrt{2}] = \frac{4}{3}[5\sqrt{10} - 4]$ sq. units	½ **2** **1 ½** **1**
35.	Let the events be E_1 : User uses Alpha smartwatch E_2 : User uses Beta smartwatch E_3 : User uses Gamma smartwatch E_4 : User uses Delta smartwatch A : User gets a false heart-rate alert Clearly $\{E_1, E_2, E_3, E_4\}$ is a partition of the sample space. Then $P(E_1)$=0.45, $P(E_2) = 0.3, P(E_3) = 0.15, P(E_4) = 0.1$ Now $P(A\|E_1) = 0.01,\ P(A\|E_2) = 0.02, P(A\|E_3) = 0.05, P(A\|E_4) = 0.01$ (i) By Theorem of Total Probability, The total probability that a randomly selected user gets a false alert $= P(A)$ $= P(E_1)P(A\|E_1) + P(E_2)P(A\|E_2) + P(E_3)P(A\|E_3) + P(E_4)P(A\|E_4)$ $= 0.45 \times 0.01 + 0.3 \times 0.02 + 0.15 \times 0.05 + 0.1 \times 0.01$ $= 0.0045 + 0.006 + 0.0075 + 0.001 = 0.019$ (ii) By Bayes Theorem, The probability that false alert came from a Beta smartwatch $= P(E_2\|A)$ $= \frac{P(E_2)P(A\|E_2)}{P(E_1)P(A\|E_1)+P(E_2)P(A\|E_2)+P(E_3)P(A\|E_3)+P(E_4)P(A\|E_4)}$ $= \frac{0.3\times0.02}{0.019} = \frac{0.006}{0.019} = \frac{6}{19}$	**1** **1** **1 ½** **1 ½**

Section –E

[This section comprises solution of 3 case- study/passage-based questions of 4 marks]

36.	(i) Given lines are $l_1: \vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{a} + \lambda\,\vec{b}$ and $l_2: \vec{r} = 2\hat{\imath} - \hat{\jmath} + \hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + \hat{k}) \equiv \vec{c} + \mu\,\vec{d}$ Here $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k},\ \vec{b} = \hat{\imath} + 3\hat{\jmath} - \hat{k},\ \vec{c} = 2\hat{\imath} - \hat{\jmath} + \hat{k},\ \vec{d} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ For direction vectors $\vec{b}$ and $\vec{d}$, $\vec{b} \neq \lambda\,\vec{d}$, for any $\lambda \in R$ $\Rightarrow$ $\vec{b}$ is not parallel to $\vec{d}$ So, lines are not parallel. (ii) Given lines are $l_1: \vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{a} + \lambda\,\vec{b}$ and $l_3: \vec{r} = 3\hat{\imath} - \hat{\jmath} + \hat{k} + \gamma(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{e} + \gamma\,\vec{f}$	**1** **1**

	Since $\vec{b} = \vec{f}$, the lines l_1 and l_3 are parallel.	
	(iii) (a) $\vec{c} - \vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k} - (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = \hat{\imath} - 3\hat{\jmath} - 2\hat{k}$ $\vec{b} \times \vec{d} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 3 & -1 \\ 2 & 3 & 1 \end{vmatrix} = 6\hat{\imath} - 3\hat{\jmath} - 3\hat{k} = 3(2\hat{\imath} - \hat{\jmath} - \hat{k})$ $(\vec{c} - \vec{a}).(\vec{b} \times \vec{d}) = (\hat{\imath} - 3\hat{\jmath} - 2\hat{k}).3(2\hat{\imath} - \hat{\jmath} - \hat{k}) = 3(2 + 3 + 2) = 21 \neq 0$ So, the lines are skew lines. $\lvert\vec{b} \times \vec{d}\rvert = 3\sqrt{2^2 + (-1)^2 + (-1)^2} = 3\sqrt{6}$ Thus, the shortest distance between the given lines is $D = \left\lvert \frac{(\vec{c} - \vec{a}).(\vec{b} \times \vec{d})}{\lvert\vec{b} \times \vec{d}\rvert} \right\rvert = \left\lvert \frac{21}{3\sqrt{6}} \right\rvert = \frac{7}{\sqrt{6}} = \frac{7\sqrt{6}}{6}\ units$	**1** **1**
	OR (b) $l_1 : \vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{a} + \lambda\,\vec{b}$ and $l_3 : \vec{r} = 3\hat{\imath} - \hat{\jmath} + \hat{k} + \gamma(\hat{\imath} + 3\hat{\jmath} - \hat{k}) \equiv \vec{e} + \gamma\,\vec{b}$ Since the lines are parallel, the shortest distance = $\frac{\lvert\vec{b} \times (\vec{e} - \vec{a})\rvert}{\lvert\vec{b}\rvert}$ $= \frac{1}{\sqrt{11}} \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 3 & -1 \\ 2 & -3 & -2 \end{vmatrix} = \frac{\lvert -9\hat{\imath} - 9\hat{k}\rvert}{\sqrt{11}} = \frac{9\sqrt{22}}{11} units$	**1** **1**
37.	(i) $W(x) = 35 + 1.6x - 0.02x^2$ $W'(x) = 1.6 - 0.04x = 0 \Rightarrow x = 40$ Therefore, the value of x is 40 crores at the stationary point of $W(x)$.	**1**
	(ii) $W''(x) = -0.04 < 0 \Rightarrow W(x)$ is maximum at $x = 40$ by the second derivative test. Thus, the WQI is maximum when ₹ 40 crores are spent on the revival. Max. WQI = $W(40) = 35 + 64 - 32 = 67$	**½** **½**
	(iii) (a) If the investment is ₹35 crores, then the WQI = $W(35)$ $W(35) = 35 + (1.6)(35) - (0.02)(35)^2$ $= (2.6)(35) - (0.02)(1225)$ $= 91 - 24.5 = 66.5$ Also, 67-66.5 = 0.5 So, the WQI is short by 0.5 units of its maximum level. **OR** (b) $W(60) = 35 + 96 - 72 = 59$ 59 is less than the maximum value 67 of WQI, even when the investment has been increased from 40 crores to 60 crores. This shows that the returns are diminishing.	**1** **1** **1** **1**

38.	(i) Taking price of each kg of apples, oranges and pomegranate as ₹x, ₹y, ₹z respectively. Matrix form is $AX = B$, where $A = \begin{pmatrix} 5 & 10 & 5 \\ 10 & 10 & 5 \\ 8 & 15 & 7 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 2750 \\ 3500 \\ 4100 \end{pmatrix}$ Here $\lvert A \rvert = 25 \neq 0$ So, the system is consistent and has unique solution Adjoint of the matrix A = adjA = $\begin{pmatrix} -5 & 5 & 0 \\ -30 & -5 & 25 \\ 70 & 5 & -50 \end{pmatrix}$ (ii) Unique solution of the system is given by $X = A^{-1}B = \frac{1}{25}\begin{pmatrix} -5 & 5 & 0 \\ -30 & -5 & 25 \\ 70 & 5 & -50 \end{pmatrix}\begin{pmatrix} 2750 \\ 3500 \\ 4100 \end{pmatrix}$ $\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 150 \\ 100 \\ 200 \end{pmatrix} \Rightarrow x = 150, y = 100, z = 200$ So the price of Apples, Oranges and Pomegranate are respectively ₹150 per kg, ₹100 per kg, ₹200 per kg.	**1** **1** **1** **1**

MATHEMATICS – Code No. 041
SAMPLE QUESTION PAPER
CLASS - XII (2025-26)

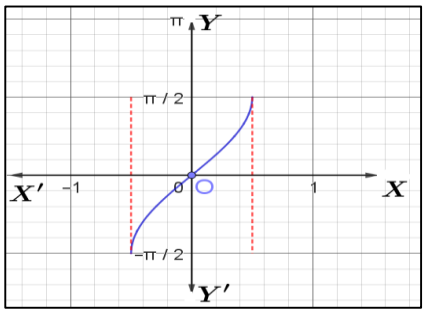
Maximum Marks: 80 **Time: 3 hours**

General Instructions:

Read the following instructions very carefully and strictly follow them:

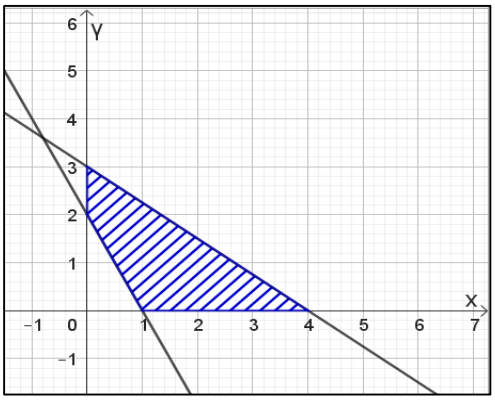
1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) with only one correct option and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculator is not allowed.

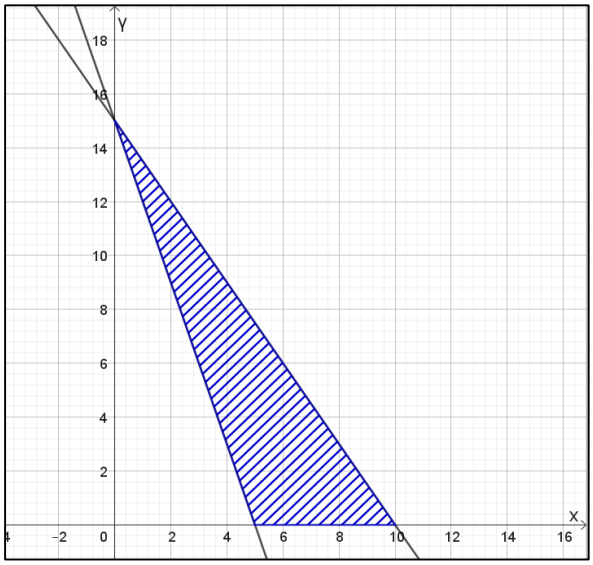
SECTION-A
This section comprises of multiple choice questions (MCQs) of 1 mark each.
Select the correct option (Question 1 - Question 18)

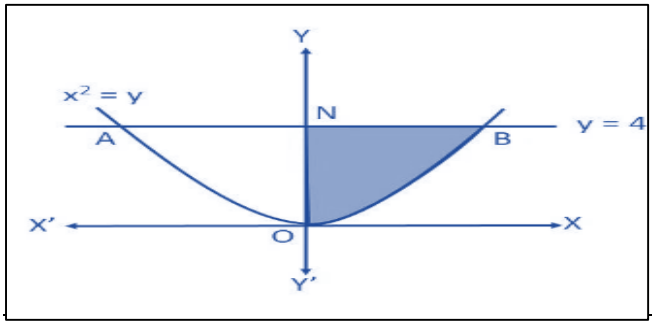
Q.No.	Questions	Marks
1.	Identify the function shown in the graph  (A) $\sin^{-1} x$ (B) $\sin^{-1}(2x)$ (C) $\sin^{-1}\left(\frac{x}{2}\right)$ (D) $2\sin^{-1} x$	**1**
	For Visually Impaired:	
1.	Inverse Trigonometric Function, whose domain is $\left[-\frac{1}{3},\frac{1}{3}\right]$, is ... (A) $\cos^{-1} x$ (B) $\cos^{-1}\left(\frac{x}{3}\right)$ (C) $\cos^{-1}(3x)$ (D) $3\cos^{-1} x$	

*Please note that the assessment scheme of the Academic Session 2024-25 will continue in the current session i.e. 2025-26.

2.	If for three matrices $A = [a_{ij}]_{m\times 4}$, $B = [b_{ij}]_{n\times 3}$ and $C = [c_{ij}]_{p\times q}$ products AB and AC both are defined and are square matrices of same order, then value of m, n, p and q are: (A) $m = q = 3$ $and\, n = p = 4$ (B) $m = 2, q = 3$ $and\, n = p = 4$ (C) $m = q = 4$ $and\, n = p = 3$ (D) $m = 4, p = 2$ $and\, n = q = 3$	1
3.	If the matrix $A = \begin{bmatrix} 0 & r & -2 \\ 3 & p & t \\ q & -4 & 0 \end{bmatrix}$ is skew-symmetric, then value of $\frac{q+t}{p+r}$ is…. (A)-2 (B) 0 (C) 1 (D) 2	1
4.	If A is a square matrix of order 4 and $\lvert adj\ A\rvert = 27$, then $A\ (adj\ A)$ is equal to (A) 3 (B) 9 (C) 3 I (D) 9 I	1
5.	The inverse of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is… (A) $\begin{bmatrix} 0 & 0 & 3 \\ 0 & 2 & 0 \\ 5 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (C) $\begin{bmatrix} -\frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{5} \end{bmatrix}$ (D) $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -5 \end{bmatrix}$	1
6.	Value of the determinant $\begin{vmatrix} \cos 67^o & \sin 67^o \\ \sin 23^o & \cos 23^o \end{vmatrix}$ is (A) 0 (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1	1
7.	If a function defined by $f(x) = \begin{cases} kx + 1, & x \le \pi \\ \cos x, & x > \pi \end{cases}$ is continuous at $x = \pi$, then the value of k is (A) π (B) $\frac{-1}{\pi}$ (C) 0 (D) $\frac{-2}{\pi}$	1
8.	If $f(x) = x\tan^{-1}x$, then $f'(1)$is equal to (A)$\frac{\pi}{4} - \frac{1}{2}$ (B)$\frac{\pi}{4} + \frac{1}{2}$ (C)$-\frac{\pi}{4} - \frac{1}{2}$ (D)$-\frac{\pi}{4} + \frac{1}{2}$	1
9.	A function $f(x) = 10 - x - 2x^2$ is increasing on the interval (A) $\left(-\infty, -\frac{1}{4}\right]$ (B) $\left(-\infty, \frac{1}{4}\right)$ (C) $\left[-\frac{1}{4}, \infty\right)$ (D)$\left[-\frac{1}{4}, \frac{1}{4}\right]$	1
10.	The solution of the differential equation $xdx + ydy = 0$ represents a family of (A) straight lines (B) parabolas (C) Circles (D) Ellipses	1

11.	If $f(a+b-x) = f(x)$, then $\int_a^b x\, f(x)dx$ is equal to (A) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (B) $\frac{a+b}{2}\int_a^b f(a-x)dx$ (C) $\frac{b-a}{2}\int_a^b f(x)\ dx$ (D) $\frac{a+b}{2}\int_a^b f(x)dx$	1
12.	If $\int x^3 \sin^4(x^4)\cos(x^4)\ dx$ = $a\ \sin^5(x^4)$ + C, then a is equal to (A) $-\frac{1}{10}$ (B) $\frac{1}{20}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$	1
13.	A bird flies through a distance in a straight line given by the vector $\hat{\imath}+2\hat{\jmath}+\hat{k}$. A man standing beside a straight metro rail track given by $\vec{r} = (3+\lambda)\hat{\imath}+(2\lambda-1)\hat{\jmath}+3\lambda\hat{k}$ is observing the bird. The projected length of its flight on the metro track is (A) $\frac{6}{\sqrt{14}}$ units (B) $\frac{14}{\sqrt{6}}$ units (C) $\frac{8}{\sqrt{14}}$ units (D) $\frac{5}{\sqrt{6}}$ units	1
14.	The distance of the point with position vector $3\hat{\imath}+4\hat{\jmath}+5\hat{k}$ from the y-axis is (A) 4 units (B) $\sqrt{34}$ units (C) 5 units (D) $5\sqrt{2}$ units	1
15.	If $\vec{a} = 3\hat{\imath}+2\hat{\jmath}+4\hat{k}$, $\vec{b} = \hat{\imath}+\hat{\jmath}-3\hat{k}$ and $\vec{c} = 6\hat{\imath}-\hat{\jmath}+2\hat{k}$ are three given vectors, then $(2\vec{a}.\hat{\imath})\hat{\imath}-(\vec{b}.\hat{\jmath})\hat{\jmath}+(\vec{c}.\hat{k})\hat{k}$ is same as the vector (A) $\vec{a}$ (B) $\vec{b}+\vec{c}$ (C) $\vec{a}-\vec{b}$ (D) $\vec{c}$	1
16.	A student of class XII studying Mathematics comes across an incomplete question in a book. Maximise $Z = 3x+2y+1$ Subject to the constraints $x \geq 0, y \geq 0, 3x+4y \leq 12$, He/ She notices the below shown graph for the said LPP problem, and finds that a constraint is missing in it: Help him/her choose the required constraint from the graph.  The missing constraint is (A) $x+2y \leq 2$ (B) $2x+y \geq 2$ (C) $2x+y \leq 2$ (D) $x+2y \geq 2$	1

16.	**For Visually Impaired:** If $Z = ax + by + c$, where $a, b, c > 0$, attains its maximum value at two of its corner points (4,0) and (0,3) of the feasible region determined by the system of linear inequalities, then (A) $4a = 3b$ (B) $3a = 4b$ (C) $4a + c = 3b$ (D) $3a + c = 4b$	
17.	The feasible region of a linear programming problem is bounded but the objective function attains its minimum value at more than one point. One of the points is (5,0).  Then one of the other possible points at which the objective function attains its minimum value is (A) (2,9) (B) (6,6) (C) (4,7) (D) (0,0) **For Visually Impaired:** The graph of the inequality $3x + 5y < 10$ is the (A) Entire XY −plane (B) Open Half plane that doesn't contain origin (C) Open Half plane that contains origin, but not the points of the line $3x + 5y = 10$ (D) Half plane that contains origin and the points of the line $3x + 5y = 10$	1
18.	A person observed the first 4 digits of your 6-digit PIN. What is the probability that the person can guess your PIN? (A) $\frac{1}{81}$ (B) $\frac{1}{100}$ (C) $\frac{1}{90}$ (D) 1	1

	ASSERTION-REASON BASED QUESTIONS **(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)** (A) Both (A) and (R) are true and (R) is the correct explanation of (A). (B) Both (A) and (R) are true but (R) is not the correct explanation of (A). (C) (A) is true but (R) is false. (D) (A) is false but (R) is true.	
19.	**Assertion (A):** Value of the expression $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1} 1 - \sec^{-1}(\sqrt{2})$ is $\frac{\pi}{4}$. **Reason (R):** Principal value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and that of $sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.	1
20.	**Assertion(A):** Given two non-zero vectors $\vec{a}$ and $\vec{b}$. If $\vec{r}$ is another non-zero vector such that $\vec{r} \times (\vec{a} + \vec{b}) = \vec{0}$. Then $\vec{r}$ is perpendicular to $\vec{a} \times \vec{b}$. **Reason (R):** The vector $(\vec{a} + \vec{b})$ is perpendicular to the plane of $\vec{a}$ and $\vec{b}$	1
SECTION B **This section comprises of 5 very short answer (VSA) type questions of 2 marks each.**		
21A	Evaluate $\tan\left(\tan^{-1}(-1) + \frac{\pi}{3}\right)$ **OR**	2
21B	Find the domain of $\cos^{-1}(3x - 2)$	
22	If $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, then prove that $\frac{dy}{dx} - \sec x = 0$	2
23A	Find: $\int \frac{(x-3)}{(x-1)^3} e^x \, dx$ **OR**	2
23B	Find out the area of shaded region in the enclosed figure. 	

23 B	**For Visually Impaired:** Find out the area of the region enclosed by the curve $y^2 = x$, $x = 3$ and x-axis in the first quadrant.	
24.	If $f(x+y) = f(x)f(y)$ for all $x, y \in$ R and $f(5)$= 2, $f'(0) = 3$, then using the definition of derivatives, find $f'(5)$.	2
25.	The two vectors $\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\widehat{3\imath} - \hat{\jmath} + 3\hat{k}$ represent the two sides OA and OB, respectively of a ΔOAB, where O is the origin. The point P lies on AB such that OP is a median. Find the area of the parallelogram formed by the two adjacent sides as OA and OP.	2
	SECTION C **This section comprises of 6 short answer (SA) type questions of 3 marks each.**	
26A.	If $x^y = e^{x-y}$ prove that $\frac{dy}{dx} = \frac{\log x}{(\log(xe))^2}$ and hence find its value at $x = e$.	3
	OR	
26B.	If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ find $\frac{d^2y}{dx^2}$.	
27	A spherical ball of ice melts in such a way that the rate at which its volume decreases at any instant is directly proportional to its surface area. Prove that the radius of the ice ball decreases at a constant rate.	3
28A	Sketch the graph $y = \|x+1\|$. Evaluate $\int_{-4}^{2} \|x+1\| dx$. What does the value of this integral represent on the graph?	3
	OR	
28B	Using integration find the area of the region $\{(x,y) : x^2 - 4y \leq 0, y - x \leq 0\}$	
	For Visually Impaired:	
28A	Define the function $y = \|x+1\|$. Evaluate $\int_{-4}^{2} \|x+1\| dx$. What does the value of this integral represent?	
	OR	
28B	Using integration find the area enclosed within the curve: $25x^2 + 16y^2 = 400$	
29A	Find the distance of the point $(2,-1,3)$ from the line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \mu(3\hat{\imath} + 6\hat{\jmath} + 2\hat{k})$ measured parallel to the z-axis.	3
	OR	
29B	Find the point of intersection of the line $\vec{r} = (3\hat{\imath} + \hat{k}) + \mu(\hat{\imath} + \hat{\jmath} + \hat{k})$ and the line through $(2,-1,1)$ parallel to the z-axis. How far is this point from the z-axis?	

30.	Solve graphically: Maximise $Z = 2x + y$ subject to $x + y \leq 1200$ $x + y \geq 600$ $y \leq \frac{x}{2}$ $x \geq 0, y \geq 0$.	3
30	**For Visually Impaired:** The objective function $Z = 3x + 2y$ of a linear programming problem under some constraints is to be maximized and minimized. The corner points of the feasible region are $A(600,0), B(1200,0), C(800, 400)$ and $D(400, 200)$. Find the point at which Z is maximum and the point at which Z is minimum. Also, find the corresponding maximum and minimum values of Z.)	
31.	Two students Mehul and Rashi are seeking admission in a college. The probability that Mehul is selected is 0.4 and the probability of selection of exactly one of the them is 0.5. Chances of selection of them is independent of each other. Find the chances of selection of Rashi. Also find the probability of selection of at least one of them.	3
	SECTION D **This section comprises of 4 long answer (LA) type questions of 5 marks each**	
32.	For two matrices $A = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ -2 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$, find the product AB and hence solve the system of equations: $3x - 6y - z = 3$ $2x - 5y - z + 2 = 0$ $-2x + 4y + z = 5$	5
33A	Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2}\, dx$ **OR**	5
33B	Find $\int \frac{(3 \sin\theta - 2)\cos\theta}{5 - cos^2\theta - 4\sin\theta}\, d\theta$	
34A	Solve the differential equation: $y + \frac{d}{dx}(xy) = x(\sin x + x)$ **OR**	5
34B	Find the particular solution of the differential equation: $2y\, e^{x/y} dx + (y - 2x\, e^{x/y})\, dy = 0$ given that $y(0) = 1$	

35.	The two lines $\frac{x-1}{3} = -y, z+1=0$ and $\frac{-x}{2} = \frac{y+1}{2} = z+2$ intersect at a point whose y-coordinate is 1. Find the co-ordinates of their point of intersection. Find the vector equation of a line perpendicular to both the given lines and passing through this point of intersection.	5

SECTION- E

This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each

36. **Case Study -1** 4

A city's traffic management department is planning to optimize traffic flow by analyzing the connectivity between various traffic signals. The city has five major spots labelled $A, B, C, D, and\ E$.



The department has collected the following data regarding one-way traffic flow between spots:

1. Traffic flows from A to B, A to C, and A to D.
2. Traffic flows from B to C and B to E.
3. Traffic flows from C to E.
4. Traffic flows from D to E and D to C.

The department wants to represent and analyze this data using relations and functions. Use the given data to answer the following questions:

I. Is the traffic flow reflexive? Justify. [1]

II. Is the traffic flow transitive? Justify. [1]

III A. Represent the relation describing the traffic flow as a set of ordered pairs. Also state the domain and range of the relation.

OR

III B. Does the traffic flow represent a function? Justify your answer. [2]

37.	**Case Study -2**	4

LED bulbs are energy-efficient because they use significantly less electricity than traditional bulbs while producing the same amount of light. They convert more energy into light rather than heat, reducing waste. Additionally, their long lifespan means fewer replacements, saving resources and money over time.

A company manufactures a new type of energy-efficient LED bulb. The cost of production and the revenue generated by selling x bulbs (in an hour) are modelled as

$C(x) = 0.5x^2 - 10x + 150$ and $R(x) = -0.3x^2 + 20x$ respectively, where $C(x)$ and $R(x)$ are both in ₹.



To maximize the profit, the company needs to analyze these functions using calculus. Use the given models to answer the following questions:

I. Derive the profit function $P(x)$ [1]

II. Find the critical points of $P(x)$. [1]

III A. Determine whether the critical points correspond to a maximum or a minimum profit by using the second derivative test.

OR

III B. Identify the possible practical value of x (i.e., the number of bulbs that can realistically be produced and sold) that can maximize the profit, if the resources available and the expenditure on machines allows to produce minimum 10 but not more than 18 bulbs per hour. Also calculate the maximum profit. [2]

38.	**Case Study -3** Excessive use of screens can result in vision problems, obesity, sleep disorders, anxiety, low retention problems and can impede social and emotional comprehension and expression. It is essential to be mindful of the amount of time we spend on screens and to reduce our screen-time by taking regular breaks, setting time limits, and engaging in non-screen-based activities.  In a class of students of the age group 14 to 17, the students were categorised into three groups according to a feedback form filled by them. The first group constituted of the students who spent more than 4 hours per day on the mobile screen or the gaming screens, while the second group spent 2 to 4 hours /day on the same activities. The third group spent less than 2 hours /day on the same. The first group with the high screen time is 60% of all the students, whereas the second group with moderate screen time is 30% and the third group with low screen time is only 10% of the total number of students. It was observed that 80% students of first group faced severe anxiety and low retention issues, with 70% of second group, and 30% of third group having the same symptoms. I. What is the total percentage of students who suffer from anxiety and low retention issues in the class? [2] II. A student is selected at random, and he is found to suffer from anxiety and low retention issues. What is the probability that he/she spends screen time more than 4 hours per day? [2]	4

MATHEMATICS – Code No. 041
MARKING SCHEME
CLASS – XII (2025-26)

SECTION-A (MCQs of 1 mark each)		
Sol. N.	**Hint / Solution**	**Marks**
1	Clearly from the graph Domain is $\left[-\frac{1}{2},\frac{1}{2}\right]$ So graph is of the function $\sin^{-1}(2x)$ **Answer is (B)** $\sin^{-1}(2x)$	**1**
1 (V.I.)	Domain is $\left[-\frac{1}{3},\frac{1}{3}\right]$ So the function is $\cos^{-1}(3x)$ **Answer is (C)** $\cos^{-1}(3x)$	**1**
2	AB is defined so n=4 AC is defined so p=4 AB and AC are square matrices of same order so $m \times 3 = m \times q \Rightarrow q = 3 = m$ **Answer is (A)** $m = q = 3 \;\; and \; n = p = 4$	**1**
3	As A is skew symmetric So $p = 0, q = 2, r = -3, t = 4$ So $\frac{q+t}{p+r} = \frac{6}{-3} = -2$ **Answer is (A) -2**	**1**
4	$\lvert adj\ A\rvert = 27 \Rightarrow \lvert A\rvert^3 = 27 = 3^3 \Rightarrow \lvert A\rvert = 3$ $A\ (adj\ A) = \lvert A\rvert\ I = 3\ I$ **Answer is (C) 3 *I***	**1**
5	Inverse of the matrix $\begin{bmatrix}3 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 5\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & 0 & 0\\ 0 & \frac{1}{2} & 0\\ 0 & 0 & \frac{1}{5}\end{bmatrix}$ **Answer is (B)**	**1**
6	$\begin{vmatrix}\cos 67^o & \sin 67^o\\ \sin 23^o & \cos 23^o\end{vmatrix} = \cos 67^o \cos 23^o - \sin 67^o \sin 23^o = \cos(67^o + 23^o) = \cos 90^o = 0$ **Answer is (A) 0**	**1**
7	$f(x)$ is continuous at $x = \pi$ $\Rightarrow \lim_{x\to\pi^-}(kx+1) = \lim_{x\to\pi^+} cos\ x = f(\pi)$ $\Rightarrow \lim_{h\to 0}[k(\pi - h) + 1] = \lim_{h\to 0} cos(\pi + h) = k\pi + 1$ $\Rightarrow k\pi + 1 = -1 \qquad \Rightarrow k = \frac{-2}{\pi}$ **Answer is (D)** $\frac{-2}{\pi}$	**1**

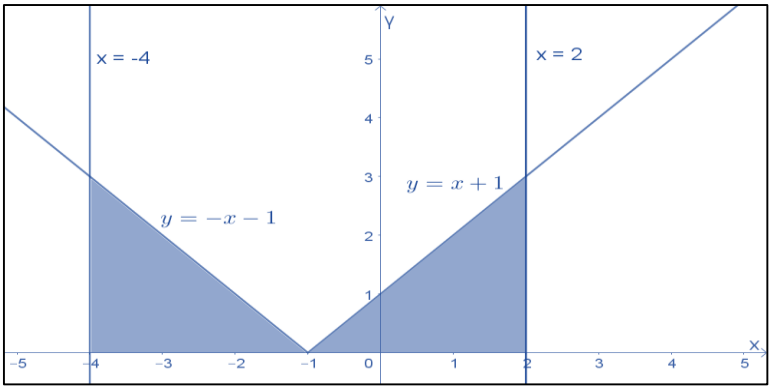
8	$f(x) = x\tan^{-1}x$ $f'(x) = 1.\tan^{-1}x + x.\frac{1}{1+x^2}$ $f'(1) = 1.\tan^{-1}1 + \frac{1}{1+1} = \frac{\pi}{4} + \frac{1}{2}$ **Answer is (B) $\frac{\pi}{4} + \frac{1}{2}$**	**1**
9	$f(x) = 10 - x - 2x^2$ $\Rightarrow f'(x) = -1 - 4x$ For increasing function $f'(x) \geq 0$ $\Rightarrow -(1 + 4x) \geq 0$ $\Rightarrow (1 + 4x) \leq 0$ $\Rightarrow x \leq -1/4$ $\Rightarrow x \in \left(-\infty, -\frac{1}{4}\right]$ **Answer is (A) $\left(-\infty, -\frac{1}{4}\right]$**	**1**
10	$xdx + ydy = 0$ $\Rightarrow \int xdx = -\int ydy$ $\Rightarrow \frac{x^2}{2} = -\frac{y^2}{2} + k$ $\Rightarrow x^2 + y^2 = 2k$ Solution is $x^2 + y^2 = 2k$, k being an arbitrary constant. **Answer is (C) Circles**	**1**
11	$I = \int_a^b x\,f(x)dx = \int_a^b (a + b - x)f(a + b - x)dx$ $\Rightarrow I = \int_a^b (a + b - x)f(x)dx$ (given $f(a + b - x) = f(x)$) $\Rightarrow I = \int_a^b (a + b)\,f(x)dx - \int_a^b x\,f(x)dx$ $\Rightarrow 2\,I = (a + b)\int_a^b f(x)dx$ $\Rightarrow I = \frac{1}{2}(a + b)\int_a^b f(x)dx$ **Answer is (D) $\frac{a+b}{2}\int_a^b \boldsymbol{f(x)dx}$**	**1**
12	Let $I = \int x^3 sin^4(x^4)\cos(x^4)\,dx$ Let $\sin(x^4) = t \Rightarrow 4x^3\cos(x^4)\,dx = dt \Rightarrow x^3\cos(x^4) = \frac{1}{4}\,dt$ Thus $I = \int t^4\,(\frac{1}{4}\,dt\,) = \frac{1}{20}\,t^5$ + C = $\frac{1}{20}\,sin^5(x^4)$ + C $\Rightarrow I = \frac{1}{20}\,sin^5(x^4) + C = a\ sin^5(x^4)$ + C So, $a = \frac{1}{20}$ **Answer is (B) $\frac{1}{20}$**	**1**
13	The projection of the vector $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ on the line $\vec{r} = (3\hat{\imath} - \hat{\jmath}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ is $\frac{1x1 + 2x2 + 1x3}{\sqrt{1^2+2^2+3^2}} = \frac{8}{\sqrt{14}}$ units **Answer is (C) $\frac{8}{\sqrt{14}}$ units**	**1**

14	The distance of the point (a, b, c) from the y-axis is $\sqrt{a^2+c^2}$ So, the distance is $\sqrt{3^2+5^2}$ = $\sqrt{34}$ units. **Answer is (B) $\sqrt{34}$ units**	**1**
15	$(2\vec{a}.\hat{\imath})\hat{\imath} - (\vec{b}.\hat{\jmath})\hat{\jmath} + (\vec{c}.\hat{k})\hat{k} = (2\times 3)\hat{\imath} - (1)\hat{\jmath} + (2)\hat{k}$ $= 6\hat{\imath} - \hat{\jmath} + 2\hat{k} = \vec{c}$ **Answer is (D) $\vec{c}$**	**1**
16	The points (1,0) and (0,2) satisfy the equation $2x + y = 2$ And shaded region shows that (0,0) doesn't lie in the feasible solution region So, the inequality is $2x + y \geq 2$ **Answer is (B) $2x + y \geq 2$**	**1**
16 (V.I.)	(4,0) and (0,3) gives maximum value so $Z_{(4,0)} = Z_{(0,3)} \Rightarrow 4a + c = 3b + c \Rightarrow 4a = 3b$ **Answer is (A) $4a = 3b$**	**1**
17	The student may read the point (2,9) from the line on the graph. The student may find the equation $3x + y = 15$ joining (5,0) and (0,15) and then verify the point (2,9) satisfies it. **Answer is (A) (2,9)**	**1**
17 (V.I.)	**Answer is (C)** Open Half plane that contains origin, but not the points of the line $3x + 5y = 10$	**1**
18	**Answer is (B) $\frac{1}{100}$** The person knows the first 4 digits. So the person has to guess the remaining two digits. P (guessing the PIN)=1×1×1×1×$\frac{1}{10}\times\frac{1}{10} = \frac{1}{100}$	**1**
19	$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1}1 - \sec^{-1}(\sqrt{2}) = \frac{\pi}{3} + \frac{\pi}{4} - \frac{\pi}{4} = \frac{\pi}{3} \neq \frac{\pi}{4}$ So, A is false. Principal Value branch of $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and that of $sec^{-1}x$ is $[0,\pi] - \left\{\frac{\pi}{2}\right\}$. So, R is true **Answer is (D)Assertion is false, but Reason is true**	**1**
20	C. $\vec{r}\times(\vec{a}+\vec{b}) = \vec{0} \Rightarrow \vec{r}$ is parallel to $(\vec{a}+\vec{b})$ and $(\vec{a}+\vec{b})$ lies on the plane of $\vec{a}$ and $\vec{b}$. So, $\vec{r}$ is parallel to the plane of $\vec{a}$ and $\vec{b} \Rightarrow \vec{r}$ is perpendicular to $(\vec{a}\times\vec{b})$. So, Assertion is true But $(\vec{a}+\vec{b})$ lies on the plane of $\vec{a}$ and $\vec{b}$, so $(\vec{a}+\vec{b})$ is not perpendicular to the plane of $\vec{a}$ and $\vec{b}$ Therefore, Reason is false. **Answer is (C) Assertion is true, but Reason is false**	**1**

SECTION B
(VSA type questions of 2 marks each)

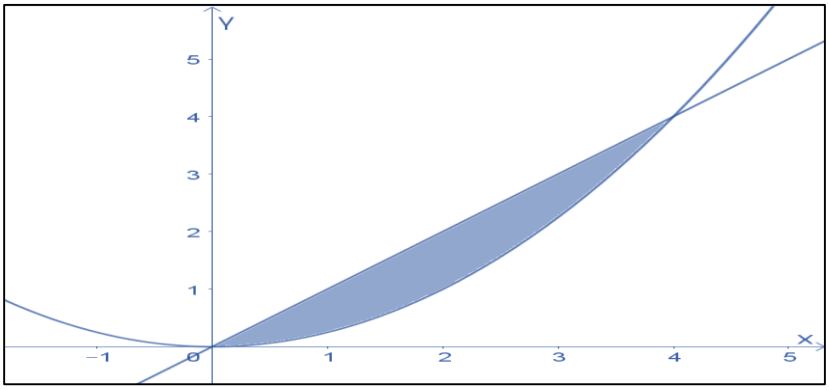
21A	$\tan\left(\tan^{-1}(-1)+\frac{\pi}{3}\right)=\tan\left(-\frac{\pi}{4}+\frac{\pi}{3}\right)$	½
	$=\frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$	1
	$=\frac{\sqrt{3}-1}{1+\sqrt{3}}$ *or* $2-\sqrt{3}$	½
	OR	OR
21B	For domain, $-1\le 3x-2\le 1$	½
	$\Rightarrow 1\le 3x\le 3$	½
	$\Rightarrow \frac{1}{3}\le x\le 1$	½
	So, domain of $\cos^{-1}(3x-2)$ is $\left[\frac{1}{3},1\right]$	½
22	$y=\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$	
	Differentiating with respect to x	
	$\frac{dy}{dx}=\frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}.sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right).\frac{1}{2}$	½
	$=\frac{\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}.\frac{1}{cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}.\frac{1}{2}$	
	$=\frac{1}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}=\frac{1}{\sin\left(\frac{\pi}{2}+x\right)}=\frac{1}{\cos x}$	1
	$\Rightarrow \frac{dy}{dx}-\sec x=0$	½
23A	$\int\frac{(x-3)e^x}{(x-1)^3}\,dx=\int\frac{(x-1-2)e^x}{(x-1)^3}\,dx$	
	$=\int\left(\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right)e^x\,dx=\int\left(\frac{1}{(x-1)^2}+\frac{d}{dx}\left(\frac{1}{(x-1)^2}\right)\right)e^x\,dx$	1
	$=\frac{e^x}{(x-1)^2}$ + c (as $\int(f(x)+f'(x))e^x\,dx=e^x\,f(x)$ + c)	1
	OR	OR
23B	A = $\int_0^4 x\,dy$ = $\int_0^4\sqrt{y}\,dy$	
	$=\frac{2}{3}\times y^{3/2}\Big]_{y=0}^{y=4}=\frac{16}{3}$ sq. units	1
		1
23B	**For Visually Impaired:**	
	A = $\int_0^3 y\,dx$ = $\int_0^3\sqrt{x}\,dx$	1
	$=\frac{2}{3}\times x^{3/2}\Big]_{x=0}^{x=3}=2\sqrt{3}$ sq. units	1

24	Given $f(x+y) = f(x)f(y)$ $f(0+5) = f(0)f(5)$ $\Rightarrow \quad f(0) = 1$ $f'(5) = \lim_{h\to 0} \frac{f(5+h)-f(5)}{h} = \lim_{h\to 0} \frac{f(5)f(h)-f(5)}{h}$ $[\because f(x+y) = f(x)f(y)]$ $= \lim_{h\to 0} \frac{2f(h)-2}{h}$ $[\because f(5) = 2]$ $= 2 \lim_{h\to 0} \frac{f(h)-1}{h} = 2 \lim_{h\to 0} \frac{f(h)-f(0)}{h} = 2f'(0)$ $= 2(3)$ $[\because f'(0) = 3]$ $= 6$	½ 1 ½
25	The vector $\overrightarrow{OP} = \frac{1}{2}(4\hat{\imath} + 4\hat{k}) = 2\hat{\imath} + 2\hat{k}$ Area of the parallelogram formed by the two adjacent sides as OA and OP $= \left\lvert(\overrightarrow{OA} \times \overrightarrow{OP})\right\rvert = \left\lVert\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 0 & 2 \end{vmatrix}\right\rVert$ $= \lvert 2\hat{\imath} - 2\hat{k}\rvert$ $= 2\sqrt{2}$ square units.	½ ½ ½ ½
	SECTION C **(SA type questions of 3 marks each)**	
26A	$x^y = e^{x-y}$ Taking log of both sides $y \log x = (x-y)\log e$ $y \log x + y = x$ (since $\log e = 1$) $\Rightarrow \quad y = \frac{x}{1+\log x}$ Differentiating with respect to x $\frac{dy}{dx} = \frac{(1+\log x).1 - x.\frac{1}{x}}{(1+\log x)^2}$ $= \frac{\log x}{(\log e + \log x)^2}$ $= \frac{\log x}{(\log(x\,e))^2}$ Now $\left.\frac{dy}{dx}\right]_{x=e} = \frac{\log e}{(\log e^2)^2} = \frac{1}{(2\log e)^2} = \frac{1}{2^2} = \frac{1}{4}$ (as $\log e = 1$) **Alternative Solution:** $x^y = e^{x-y}$ Taking log of both sides $y \log x = (x-y)\log e$ $y \log x + y = x$ (since $\log e = 1$) Differentiating both sides w.r.t. x $\log x \frac{dy}{dx} + \frac{y}{x} + \frac{dy}{dx} = 1$ $\Rightarrow \frac{dy}{dx}(1+\log x) = 1 - \frac{y}{x}$ $\Rightarrow \frac{dy}{dx} = \frac{x-y}{x(1+\log x)} = \frac{x-\frac{x}{1+\log x}}{x(1+\log x)} = \frac{x(1+\log x)-x}{x(1+\log x)^2} = \frac{x(1+\log x-1)}{x(\log e+\log x)^2} = \frac{\log x}{(\log(xe))^2}$ Now $\left.\frac{dy}{dx}\right]_{x=e} = \frac{\log e}{(\log e^2)^2} = \frac{1}{(2\log e)^2} = \frac{1}{2^2} = \frac{1}{4}$ (as $\log e = 1$)	1 1 1

	OR	OR
26B	$\frac{dx}{d\theta} = a(1-\cos\theta)$, $\frac{dy}{d\theta} = a(0+\sin\theta)$,	1
	$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1-\cos\theta)}$	
	$= \frac{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2sin^2\left(\frac{\theta}{2}\right)} = \cot\frac{\theta}{2}$	1
	$\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2}\,cosec^2\left(\frac{\theta}{2}\right)\frac{d\theta}{dx}$	
	$= -\frac{1}{2a}\,cosec^2\left(\frac{\theta}{2}\right)\frac{1}{2sin^2\left(\frac{\theta}{2}\right)}$	
	$= -\frac{1}{4a}\,cosec^4\left(\frac{\theta}{2}\right)$	1
27	Let r be the radius of ice ball at time t .	
	$V = \frac{4}{3}\pi r^3$..... (1)	
	S = $4\pi r^2$.....(2)	½
	Given $\frac{dV}{dt} \propto$ S	
	$\Rightarrow \frac{dV}{dt} = -$ k S (where k is some positive constant) .....(3)	½
	Differentiating (1) w.r.t. t, we get	
	$\frac{dV}{dt} = \frac{4}{3}\pi.(3r^2)\frac{dr}{dt}$	1
	$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ ..... (4)	
	$\Rightarrow$ - k S = $4\pi r^2\frac{dr}{dt}$ (from (3) and (4))	½
	$\Rightarrow$ - k S = S $\frac{dr}{dt}$ (using (2))	
	$\Rightarrow \frac{dr}{dt} = -$ k	½
	$\Rightarrow$ radius of the ice-ball decreases at a constant rate	
28A		1
	$\int_{-4}^{2}\lvert x+1\rvert dx = \int_{-4}^{-1}(-x-1)\,dx + \int_{-1}^{2}(x+1)\,dx$	½
	$= -\frac{(x+1)^2}{2}\Big]_{-4}^{-1} + \frac{(x+1)^2}{2}\Big]_{-1}^{2}$	½
	$= -\left(0-\frac{9}{2}\right)+\left(\frac{9}{2}-0\right) = 9$	½
	It represent the area of shaded region bounded by the curve $y = \lvert x+1\rvert$, $x-$ axis and the lines $x = -4\ and\ x = 2$	½

OR

28B



1

Required Area = $\int_0^4 x\,dx - \int_0^4 \frac{x^2}{4}dx$ 1

$= \frac{x^2}{2}\Big]_0^4 - \frac{1}{12}[x^3]_0^4$ ½

$= \frac{1}{2}(16-0) - \frac{1}{12}(64-0) = 8 - \frac{16}{3} = \frac{8}{3}$ sq. units ½

For Visually Impaired:

$$y = |x+1| = f(x) = \begin{cases} -x-1, & x < -1 \\ x+1, & x \geq -1 \end{cases}$$ 1

$\int_{-4}^{2}|x+1|dx = \int_{-4}^{-1}(-x-1)\,dx + \int_{-1}^{2}(x+1)\,dx$

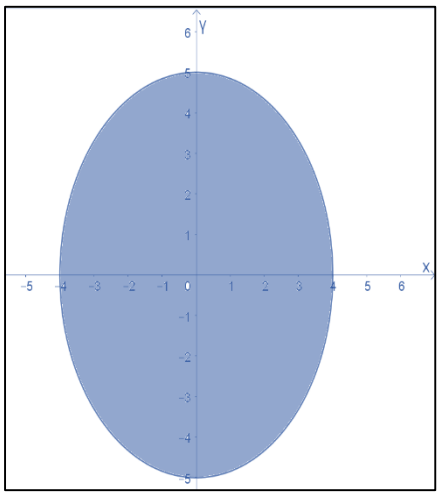
$= -\frac{(x+1)^2}{2}\Big]_{-4}^{-1} + \frac{(x+1)^2}{2}\Big]_{-1}^{2}$ 1

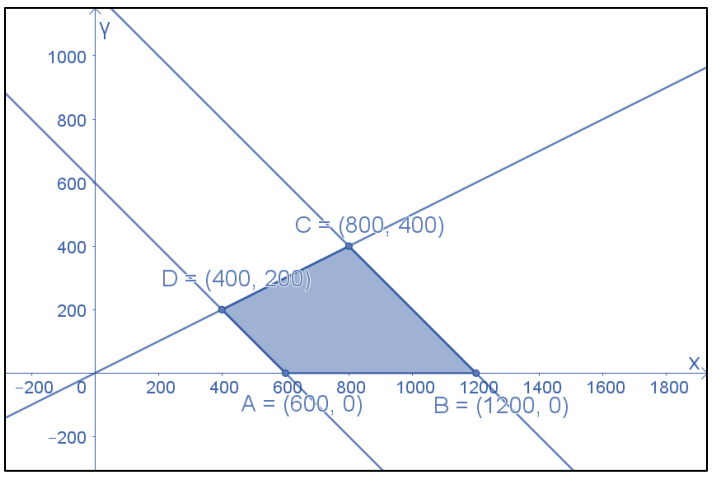
$= -\left(0-\frac{9}{2}\right) + \left(\frac{9}{2}-0\right) = 9$

It represent the area of shaded region bounded by the curve $y = |x+1|$, 1

$x-$ axis and the lines $x = -4\ and\ x = 2$

OR



	$25x^2 + 16y^2 = 400 \Rightarrow \frac{x^2}{16} + \frac{y^2}{25} = 1 \Rightarrow \frac{x^2}{4^2} + \frac{y^2}{5^2} = 1 \Rightarrow y = \frac{5}{4}\sqrt{4^2 - x^2}$ Required Area $= 4\int_0^4 \frac{5}{4}\sqrt{4^2 - x^2}\,dx$ $= 5\left[\frac{x\sqrt{4^2-x^2}}{2} + \frac{4^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$ $= 5[0 + 8\sin^{-1}(1) - 0]$ $= 40 \times \frac{\pi}{2} = 20\pi$ sq. units	1 1 1
29A	The line through $(2,-1,3)$ parallel to the z-axis is given by $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(\hat{k})$ Any point on this line is P $(2,-1,3+\lambda)$ Any point on the given line $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \mu(3\hat{\imath} + 6\hat{\jmath} + 2\hat{k})$ is Q $(2+3\mu,-1+6\mu,2+2\mu)$ For the intersection point Q $(2+3\mu,-1+6\mu,2+2\mu)$ = P (2, -1, 3+ λ) $\Rightarrow 2 = 2 + 3\mu \Rightarrow \mu = 0$ The point of intersection is $(2,-1,2)$ The distance from $(2,-1,3)$ to $(2,-1,2)$ is clearly 1 unit. **Alternative Solution:** Any point on the line through $(2,-1,3)$ parallel to the z-axis is $(2,-1,\lambda)$ Any point on the given line is $(2+3\mu,-1+6\mu,2+2\mu)$ Therefore, $2 = 2 + 3\mu \Rightarrow \mu = 0$ The point of intersection is $(2,-1,2)$ The distance from $(2,-1,3)$ to $(2,-1,2)$ is clearly 1 unit.	1 ½ ½ ½ 1 1 ½ ½
29B	**OR** The line through (2,−1,1)parallel to the z-axis is $\vec{r} = (2\hat{\imath} - \hat{\jmath} + \hat{k}) + \lambda(\hat{k})$ Any point on this line is P (2, −1, 1+ λ) Any point on the given line is A (3+ μ, μ, 1+ μ) A (3+ μ, μ, 1+ μ) = P (2, −1, 1+ λ) $\Rightarrow \mu = -1$ The point of intersection is (2, −1,0) The distance of $(2,-1,0)$ from the z-axis is $\sqrt{2^2 + (-1)^2} = \sqrt{5}$ units.	1 1 ½ ½
30	Sketching the graph  	$1\frac{1}{2}$

30	Corner points A(600,0), B(1200,0), C(800,400), D(400,200) Values of Z: $Z_A = 1200, Z_B = 2400, Z_C = 2000, Z_D = 1000$ Maximum $Z = 2400$ when $x = 1200$ and $y = 0$ **For Visually Impaired:** At Corner points A(600,0), B(1200,0), C(800,400), D(400,200) Values of Z are $Z_A = 1800, Z_B = 3600, Z_C = 3200, Z_D = 1600$ Maximum Value of Z = 3600 at B(1200,0) And Minimum Value of Z= 1600 at D(400,200)	½ ½ ½ 1 1 1
31	Let the events be: A: Mehul is selected B: Rashi is selected Then according to the question, A and B are independent events and $P(A) = 0.4, \quad P(A \cap \bar{B}) + P(B \cap \bar{A}) = 0.5$ Let $P(B) = x$ Then $P(A \cap \bar{B}) + P(B \cap \bar{A}) = 0.5$ $\Rightarrow P(A)P(\bar{B}) + P(B)P(\bar{A}) = 0.5$ $\Rightarrow 0.4(1-x) + x(1-0.4) = 0.5$ $\Rightarrow 0.4 - 0.4x + 0.6x = 0.5$ $\Rightarrow 0.2x = 0.5 - 0.4 = 0.1$ $\Rightarrow x = \frac{0.1}{0.2} = \frac{1}{2} = 0.5$ So, probability of selection of Rashi = 0.5 Probability of selection of at least one of them $= 1 - P(\bar{A} \cap \bar{B})$ $= 1 - P(\bar{A})P(\bar{B})$ $= 1 - 0.6 \times 0.5$ $= 1 - 0.3 = 0.7$	 1 1 1
	SECTION D **(LA type questions of 5 marks each)**	
32	$AB = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ -2 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ So, $A^{-1} = B \; and \; B^{-1} = A$ Given system of equations is $3x - 6y - z = 3, \;\; 2x - 5y - z + 2 = 0, \;\; -2x + 4y + z = 5$ In matrix form it can be written as: $AX = C$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \; and \; C = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$ Here $\lvert A \rvert = -3 - 0 + 2 = -1 \neq 0$ So, the system is consistent and has unique solution given by the expression $X = A^{-1}C = BC$ $\Rightarrow X = \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 21 \end{bmatrix}$ Thus $x = 2, y = -3, z = 21$	1 ½ ½ ½ ½ 1 ½

33A	Let $x = \tan\theta \Rightarrow dx = sec^2\theta\, d\theta$	½
	$I = \int_0^{\frac{\pi}{4}} \frac{\log(1+tan\theta)}{1+tan^2\theta} . sec^2\theta \; d\theta$	
	$I = \int_0^{\frac{\pi}{4}} log\,(1+tan\theta)d\theta = \int_0^{\frac{\pi}{4}} log\,\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right] d\theta$	1
	$=\int_0^{\frac{\pi}{4}} \log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right] d\theta$	1
	$=\int_0^{\frac{\pi}{4}} \log\left[\frac{1+\tan\theta+1-\tan\theta}{1+\tan\theta}\right] d\theta$	
	$=\int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1+\tan\theta}\right] d\theta$	
	$=\int_0^{\frac{\pi}{4}} \log 2 \; d\theta - \int_0^{\frac{\pi}{4}} \log[1+tan\theta] \; d\theta$	1
	$= \log 2 \times x]_0^{\frac{\pi}{4}} - I$	
	$\Rightarrow \; 2I = \frac{\pi}{4}\log 2$	1
	$\Rightarrow \; I = \frac{\pi}{8}\log 2$	½
33B	**OR**	OR
	I = $\int \frac{(3\,\sin\theta-2)\cos\theta}{5-cos^2\theta-4\sin\theta} d\theta = \int \frac{(3\,sin\theta-2)\,cos\theta}{5-(1-sin^2\theta)-4\,sin\,\theta} d\theta$	½
	$Let \quad sin\,\theta = t \implies cos\,\theta \; d\theta = dt$	
	$I = \int \frac{(3t-2)}{5-(1-t^2)-4t} dt$	1
	$= \int \frac{(3t-2)}{t^2-4t+4} dt = \int \frac{3t-2}{(t-2)^2} dt$	
	Let $\frac{3t-2}{(t-2)^2} = \frac{A}{(t-2)} + \frac{B}{(t-2)^2}$	
	$3t-2 = A(t-2)+B$	
	Comparing the coefficients of t and constant terms on both sides	½
	$A = 3, -2A+B = -2, B = 4$	+½
	$\int \frac{(3\,\sin\theta-2)\cos\theta}{5-cos^2\theta-4\sin\theta} d\theta = \int \frac{3}{t-2} dt + \int \frac{4}{(t-2)^2}$ dt	
	$= 3\log\lvert t-2\rvert - \frac{4}{t-2} + C$	1+1
	$= 3\log\lvert sin\,\theta - 2\rvert - \frac{4}{\sin\theta-2}$ + C	½
34A	$y + \frac{d}{dx}(\,xy\,) = x\,(\sin x + x)$	
	$\implies y + (x\frac{dy}{dx} + y\,) = x\,(\sin x + x)$	1
	$\implies 2y + x\frac{dy}{dx} = x\,(\sin x + x)$	
	$\implies \frac{dy}{dx} + \frac{2y}{x} = (\sin x + x)$	
	This a linear differential equation of the form $\frac{dy}{dx} + Py = Q$	
	P= $\frac{2}{x}$,Q=$(\sin x + x)$	1
	I.F =$e^{\int \frac{2}{x} dx}$= $e^{2\,log\,x}$= $e^{log\,x^2}$ = x^2	
	Solution will be y . I.F =$\int Q.IF\, dx$	1
	$yx^2 = \int (sin\,x + x)\,x^2$ dx	
	$yx^2 = \int sinx.x^2\, dx + \int x^3\, dx$	

	$\Rightarrow \quad yx^2 = -x^2\cos x + 2\int x\cos x\, dx \; + \frac{x^4}{4}$ + C	1
	$\Rightarrow yx^2 = -x^2 cos\, x$ +2 $(x\, sinx + cos\, x)$ + $\frac{x^4}{4}$ + C	
	Which is the required solution	1
	OR	
34B	$2y\, e^{x/y}dx + (\, y - 2x\, e^{x/y}\,)\, dy = 0$	
	$\Rightarrow \quad \frac{dx}{dy} = \frac{2x\, e^{x/y} - y}{2y\, e^{x/y}} = \frac{2\frac{x}{y} e^{\frac{x}{y}} - 1}{2\, e^{\frac{x}{y}}}$	1
	It is a homogeneous differential equation.	
	Let $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$	1
	$v + y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$	
	$\Rightarrow y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v = \frac{2ve^v - 1 - 2ve^v}{2e^v}$	
	$\Rightarrow \quad y\frac{dv}{dy} = \frac{-1}{2e^v}$	
	$\Rightarrow 2e^v\, dv = -\frac{dy}{y}$	1
	$\int 2e^v\, dv = -\int \frac{dy}{y}$	
	$\Rightarrow 2e^v = -\log\lvert y\rvert$ + C	
	$\Rightarrow 2e^{\frac{x}{y}} +"\, log\lvert y\rvert$ = C	1
	When $x = 0$, $y = 1$, C = 2	
	Required solution 2 $e^{\frac{x}{y}}$ + $\log\lvert y\rvert$ = 2	1
35	Let $\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda$ $\Rightarrow$ Any point on it is $(3\,\lambda + 1, -\lambda, -1)$	½
	For the point where $y = 1$ $\Rightarrow$ $\lambda = -1$	1
	$\Rightarrow$ The point is $(-2, 1, -1)$	½
	The directions of the two lines are $\vec{m} = 3\hat{\imath} - \hat{\jmath}$	1
	and $\vec{n} = -2\hat{\imath} + 2\hat{\jmath} + \hat{k}$	½
	$\vec{m} \times \vec{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -1 & 0 \\ -2 & 2 & 1 \end{vmatrix} = -\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$	1
	The equation of the required line is	
	$\vec{r} = (-2\hat{\imath} + \hat{\jmath} - \hat{k}) + \mu(-\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$	½
	Alternative Solution:	
	Let $\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda$ $\Rightarrow$ Any point on it is $(3\,\lambda + 1, -\lambda, -1)$	½
	For the point where $y = 1$ $\Rightarrow$ $\lambda = -1$	1
	$\Rightarrow$ The point is $(-2, 1, -1)$	½
	Let the direction ratios of the required line be a, b, c	
	Then $3a - b = 0$	
	And $-2a + 2b + c = 0$	1
	Solving we get $\frac{a}{-1} = \frac{-b}{3} = \frac{c}{4} \Rightarrow \frac{a}{-1} = \frac{b}{-3} = \frac{c}{4}$	1
	The required line is $\frac{x+2}{-1} = \frac{y-1}{-3} = \frac{z+1}{4} = \mu$	½
	In vector form $\vec{r} = (-2\hat{\imath} + \hat{\jmath} - \hat{k}) + \mu(-\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$	½

	SECTION- E **(3 case-study/passage-based questions of 4 marks each)**	
36	I. Traffic flow is not reflexive as $(A, A) \notin R$ (or no major spot is connected with itself) II. Traffic flow is not transitive as $(A, B) \in R \text{ and } (B, E) \in R, \text{but } (A, E) \notin R$ III A. $R = \{(A, B), (A, C), (A, D), (B, C), (B, E), (C, E), (D, E), (D, C)\}$ $\text{Domain} = \{A, B, C, D\}$ $\text{Range} = \{B, C, D, E\}$ **OR** III B. No, the traffic flow doesn't represent a function as A has three images.	1 1 1 ½ + ½ 1+1
37	I. $P(x) = R(x) - C(x) = -0.3x^2 + 20x - (0.5x^2 - 10x + 150)$ $= -0.8x^2 + 30x - 150$ II. For critical points $P'(x) = 0 \Rightarrow -1.6x + 30 = 0$ $\Rightarrow x = \frac{30}{1.6} = \frac{300}{16} = 18.75$ III A. Now $P''(x) = -1.6$ In particular $P''(18.75) = -1.6 < 0$ So, critical value $x = 18.75$ corresponds to a maximum profit. **OR** III B. As x is the number of bulbs, so practically 18 bulbs correspond to a maximum profit. Maximum profit is $P(18) = -0.8 \times 18^2 + 30 \times 18 - 150$ $= -259.2 + 540 - 150$ $= 540 - 409.2 = ₹130.80$	1 1 1 1 1 1
38	Let the events be E_1: the student is in the first group (time spent on screen is more than 4 hours) E_2: the student is in the second group (time spent on screen is 2 to 4 hours) E_3: the student is in third group (time spent on screen is less than 2 hours) A: the event of the student showing symptoms of anxiety and low retention $P(E_1) = \frac{60}{100}$ $P(E_2) = \frac{30}{100}$ and $P(E_3) = \frac{10}{100}$ $P(A\|E_1) = \frac{80}{100}$ $P(A\|E_2) = \frac{70}{100}$ and $P(A\|E_3) = \frac{30}{100}$ I. $P(A) = P(E_1) \times P(A\|E_1) + P(E_2) \times P(A\|E_2) + P(E_3) \times P(A\|E_3)$ $= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{70}{100} + \frac{10}{100} \times \frac{30}{100} = \frac{72}{100} = 72\%$ II. $P(E_1\|A) = \frac{P(E_1 \cap A)}{P(A)}$ $= \frac{\left(\frac{60}{100} \times \frac{80}{100}\right)}{\left(\frac{72}{100}\right)} = \frac{48}{72} = \frac{2}{3}$	 2 2

SAMPLE QUESTION PAPER (2024 - 25)

CLASS- XII

SUBJECT: Mathematics (041)

Time: 3 Hours **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains **38** questions. **All** questions are **compulsory.**

(ii) This Question paper is divided into **five** Sections - **A, B, C, D** and **E.**

(iii) In **Section A**, Questions no. **1** to **18** are **multiple choice questions (MCQs)** and Questions no. **19** and **20** are **Assertion-Reason based** questions of **1 mark each**.

(iv) In **Section B,** Questions no. **21** to **25** are **Very Short Answer (VSA)-type** questions, carrying **2 marks each**.

(v) In **Section C,** Questions no. **26** to **31** are **Short Answer (SA)-type** questions, carrying **3 marks each**.

(vi) In **Section D,** Questions no. **32** to **35** are **Long Answer (LA)-type** questions, carrying **5 marks each**.

(vii) In **Section E,** Questions no. **36** to **38** are **Case study-based questions,** carrying **4 marks each.**

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.

(ix) Use of calculators is **not** allowed.

SECTION-A $[1\times 20=20]$

(*This section comprises of multiple choice questions (MCQs) of 1 mark each*)

Select the correct option (Question 1 - Question 18):

Q.1. If for a square matrix A, $A.(adjA)=\begin{bmatrix}2025 & 0 & 0\\ 0 & 2025 & 0\\ 0 & 0 & 2025\end{bmatrix}$, then the value of $|A|+|adj\ A|$ is equal to:

(A) 1 (B) $2025+1$ (C) $(2025)^2+45$ (D) $2025+(2025)^2$

Q.2. Assume X, Y, Z, W and P are matrices of order $2\times n$, $3\times k$, $2\times p$, $n\times 3$ and $p\times k$, respectively. Then the restriction on n, k and p so that $PY+WY$ will be defined are:

(A) $k=3, p=n$ (B) k **is arbitrary,** $p=2$

(C) p **is arbitrary,** $k=3$ (D) $k=2, p=3$

Q.3. The interval in which the function f defined by $f(x)=e^x$ is strictly increasing, is

(A) $[1,\infty)$ (B) $(-\infty,0)$ (C) $(-\infty,\infty)$ (D) $(0,\infty)$

Q.4. If $\boldsymbol{A}$ **and** $\boldsymbol{B}$ are non-singular matrices of same order with $\boldsymbol{det(A) = 5}$, then $\boldsymbol{det\left(B^{-1}AB\right)^2}$ is equal to

(A) $\mathbf{5}$ (B) $\mathbf{5^2}$ (C) $\mathbf{5^4}$ (D) $\mathbf{5^5}$

Q.5. The value of $\boldsymbol{'n'}$, such that the differential equation $\boldsymbol{x^n \frac{dy}{dx} = y(logy - logx + 1)}$;

$\boldsymbol{(where\ x, y\ \in R^+)}$ is homogeneous, is

(A) **0** (B) **1** (C) **2** (D) **3**

Q.6. If the points $\boldsymbol{(x_1, y_1)}$, $\boldsymbol{(x_2, y_2)}$ **and** $\boldsymbol{(x_1 + x_2, y_1 + y_2)}$ are collinear, then $\boldsymbol{x_1 y_2}$ is equal to

(A) $\boldsymbol{x_2 y_1}$ (B) $\boldsymbol{x_1 y_1}$ (C) $\boldsymbol{x_2 y_2}$ (D) $\boldsymbol{x_1 x_2}$

Q.7. If $\boldsymbol{A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}}$ is a skew-symmetric matrix then the value of $\boldsymbol{a + b + c =}$

(A)1 (B) **2** (C) **3** (D) **4**

Q.8. For any two events $\boldsymbol{A}$ and $\boldsymbol{B}$, if $\boldsymbol{P\left(\overline{A}\right) = \frac{1}{2}}$, $\boldsymbol{P\left(\overline{B}\right) = \frac{2}{3}}$ and $\boldsymbol{P\left(A \cap B\right) = \frac{1}{4}}$, then $P\left(\bar{A}/\bar{B}\right)$ equals:

(A) $\frac{3}{8}$ (B) $\frac{8}{9}$ (C) $\frac{5}{8}$ (D) $\frac{1}{4}$

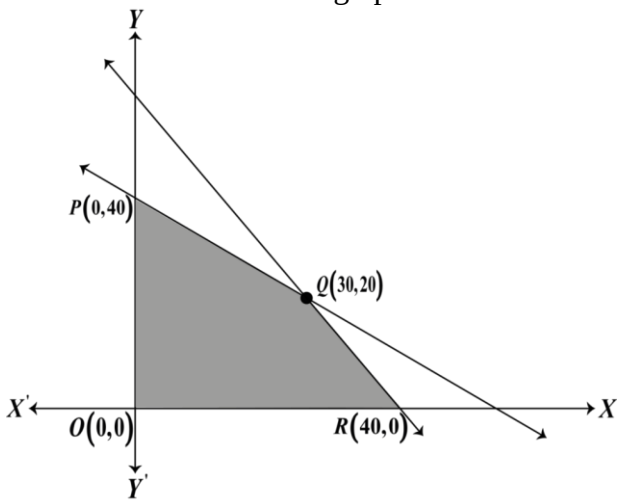
Q.9. The value of α if the angle between $\vec{p} = 2\alpha^2\hat{\imath} - 3\alpha\hat{\jmath} + \hat{k}$ and $\vec{q} = \hat{\imath} + \hat{\jmath} + \alpha\hat{k}$ is obtuse, is

(A) $R - [0, 1]$ (B) (0, 1) (C) [0, ∞) (D) [1, ∞)

Q.10. If $|\vec{a}| = 3, \left|\vec{b}\right| = 4$ and $\left|\vec{a} + \vec{b}\right|$ =5, then $\left|\vec{a} - \vec{b}\right| =$

(A) **3** (B) **4** (C) **5** (D) **8**

Q.11. For the linear programming problem (**LPP**), the objective function is $\boldsymbol{Z = 4x + 3y}$ and the feasible region determined by a set of constraints is shown in the graph:



(Note*: The figure is not to scale.)*

Which of the following statements is true?

(A) Maximum value of $\boldsymbol{Z}$ is at $\boldsymbol{R(40,0)}$.

(B) Maximum value of $\boldsymbol{Z}$ is at $\boldsymbol{Q(30,20)}$.

(C) Value of $\boldsymbol{Z}$ at $\boldsymbol{R(40,0)}$ is less than the value at $\boldsymbol{P(0,40)}$.

(D) The value of $\boldsymbol{Z}$ at $\boldsymbol{Q(30,20)}$ is less than the value at $\boldsymbol{R(40,0)}$.

Q.12. $\int \frac{dx}{x^3(1+x^4)^{\frac{1}{2}}}$ equals

(A) $-\frac{1}{2x^2}\sqrt{1+x^4}+c$ (B) $\frac{1}{2x}\sqrt{1+x^4}+c$

(C) $-\frac{1}{4x}\sqrt{1+x^4}+c$ (D) $\frac{1}{4x^2}\sqrt{1+x^4}+c$

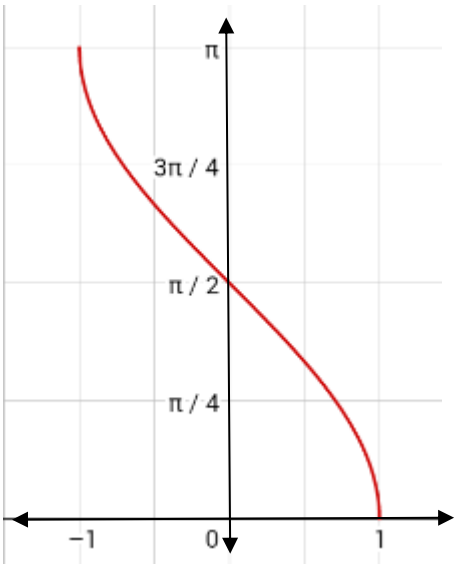
Q.13. $\int_0^{2\pi} cosec^7 x\, dx =$

(A) **0** (B) **1** (C) **4** (D) 2π

Q.14. What is the general solution of the differential equation $e^{y'} = x$?

(A) $y = xlogx + c$ (B) $y = xlogx - x + c$ (C) $y = xlogx + x + c$ (D) $y = x + c$

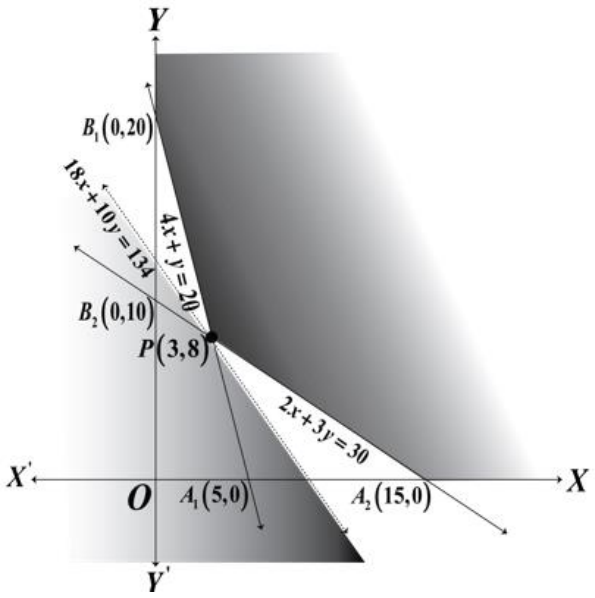
Q.15. The graph drawn below depicts



(A) y = $sin^{-1}x$ (B) y = $cos^{-1}x$ (C) y = $cos\ ec^{-1}x$ (D) y = $cot^{-1}x$

Q.16. A linear programming problem (**LPP**) along with the graph of its constraints is shown below.

The corresponding objective function is: $Z = 18x + 10y$, which has to be minimized. The smallest value of the objective function Z is 134 and is obtained at the corner point $(3,8)$,



(Note*: The figure is not to scale.)*

The optimal solution of the above linear programming problem _____.

(A) does not exist as the feasible region is unbounded.

(B) does not exist as the inequality $18x + 10y < 134$ does not have any point in common with the feasible region.

(C) exists as the inequality $18x + 10y > 134$ has infinitely many points in common with the feasible region.

(D) exists as the inequality $18x + 10y < 134$ does not have any point in common with the feasible region.

Q.17. The function $f: R \to Z$ defined by $f(x) = [x]$; where $[.]$ denotes the greatest integer function, is

(A) Continuous at $x = 2.5$ but not differentiable at $x = 2.5$

(B) Not Continuous at $x = 2.5$ but differentiable at $x = 2.5$

(C) Not Continuous at $x = 2.5$ and not differentiable at $x = 2.5$

(D) Continuous as well as differentiable at $x = 2.5$

Q.18. A student observes an open-air Honeybee nest on the branch of a tree, whose plane figure is parabolic shape given by $x^2 = 4y$. Then the area (in sq units) of the region bounded by parabola $x^2 = 4y$ and the line $y = 4$ is

(A) $\frac{32}{3}$ (B) $\frac{64}{3}$ (C) $\frac{128}{3}$ (D) $\frac{256}{3}$

ASSERTION-REASON BASED QUESTIONS

(Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.)

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

Q.19. Assertion (A): Consider the function defined as $f(x) = |x| + |x-1|,\ x \in R$. Then $\boldsymbol{f(x)}$ is not differentiable **at $x = 0$ and $x = 1$**.

Reason (R): Suppose $\boldsymbol{f}$ be defined and continuous on $\boldsymbol{(a,b)}$ and $\boldsymbol{c \in (a,b)}$, then $\boldsymbol{f(x)}$ is not differentiable at $x = c$ if $\lim_{h\to 0^-} \frac{f(c+h)-f(c)}{h} \neq \lim_{h\to 0^+} \frac{f(c+h)-f(c)}{h}$.

Q.20. Assertion (A): The function $f: R - \left\{(2n+1)\frac{\pi}{2} : n \in Z\right\} \to (-\infty, -1] \cup [1, \infty)$ defined by $\boldsymbol{f(x) = \sec x}$ is not one-one function in its domain.

Reason (R): The line $\boldsymbol{y = 2}$ meets the graph of the function at more than one point.

SECTION B $\boldsymbol{[2\times 5 = 10]}$

(This section comprises of 5 very short answer (VSA) type questions of 2 marks each.)

Q.21. If $cot^{-1}(3x+5) > \frac{\pi}{4}$, then find the range of the values of $\boldsymbol{x}$.

Q.22. The cost (in rupees) of producing $\boldsymbol{x}$ items in factory, each day is given by

$$C(x) = 0.00013x^3 + 0.002\,x^2 + 5x + 2200$$

Find the marginal cost when 150 items are produced.

Q.23. (a) Find the derivative of $\boldsymbol{\tan^{-1} x}$ with respect to $\boldsymbol{\log x}$; (where $\boldsymbol{x \in (1,\infty)}$).

OR

Q.23. (b) Differentiate the following function with respect to x : $(cos\,x)^x$; $\left(\text{where} x \in \left(0, \frac{\pi}{2}\right)\right)$.

Q.24. (a) If vectors $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{b} + \lambda\vec{c}$ is perpendicular to $\vec{a}$, then find the value of λ.

OR

Q.24. (b) A person standing at $\boldsymbol{O(0,0,0)}$ is watching an aeroplane which is at the coordinate point $\boldsymbol{A(4,0,3)}$. At the same time he saw a bird at the coordinate point $\boldsymbol{B(0,0,1)}$. Find the angles which $\overrightarrow{BA}$ makes with the x,y and z axes.

Q.25. The two co-initial adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find its diagonals and use them to find the area of the parallelogram.

SECTION C $[3\times6=18]$

(This section comprises of 6 short answer (SA) type questions of 3 marks each.)

Q.26. A kite is flying at a height of 3 metres and 5 metres of string is out. If the kite is moving away horizontally at the rate of 200 cm/s, find the rate at which the string is being released.

Q.27. According to a psychologist, the ability of a person to understand spatial concepts is given by $A=\frac{1}{3}\sqrt{t}$, where t is the age in years, $t\in[5,18]$. Show that the rate of increase of the ability to understand spatial concepts decreases with age in between 5 and 18.

Q.28. (a) An ant is moving along the vector $\overrightarrow{l_1}=\hat{\imath}-2\hat{\jmath}+3\hat{k}$. Few sugar crystals are kept along the vector $\overrightarrow{l_2}=3\hat{\imath}-2\hat{\jmath}+\hat{k}$ which is inclined at an angle θ with the vector $\overrightarrow{l_1}$. Then find the angle θ. Also find the scalar projection of $\overrightarrow{l_1}$ *on* $\overrightarrow{l_2}$.

OR

Q.28. (b) Find the vector and the cartesian equation of the line that passes through $(-1,2,7)$ and is perpendicular to the lines $\vec{r}=2\hat{\imath}+\hat{\jmath}-3\hat{k}+\lambda(\hat{\imath}+2\hat{\jmath}+5\hat{k})$ and $\vec{r}=3\hat{\imath}+3\hat{\jmath}-7\hat{k}+\mu(3\hat{\imath}-2\hat{\jmath}+5\hat{k})$.

Q.29. (a) Evaluate: $\int\left\{\frac{1}{\log x}-\frac{1}{(\log x)^2}\right\}dx$; (**where** $x>1$).

OR

Q.29. (b) Evaluate : $\int_0^1 x(1-x)^n dx$; (**where** $n\in N$).

Q.30. Consider the following Linear Programming Problem:

Minimise $Z=x+2y$

Subject to $2x+y\geq3,\ x+2y\geq6,\ x,y\geq0.$

Show graphically that the minimum of Z occurs at more than two points

Q.31. (a) The probability that it rains today is **0.4**. If it rains today, the probability that it will rain tomorrow is **0.8**. If it does not rain today, the probability that it will rain tomorrow is **0.7**. If

P_1: denotes the probability that it does not rain today.

P_2: denotes the probability that it will not rain tomorrow, if it rains today.

P_3: denotes the probability that it will rain tomorrow, if it does not rain today.

P_4: denotes the probability that it will not rain tomorrow, if it does not rain today.

(i) Find the value of $P_1\times P_4-P_2\times P_3$. $[2\ Marks]$

(ii) Calculate the probability of raining tomorrow. $[1\ Mark]$

OR

Q.31. (b) A random variable X can take all non – negative integral values and the probability that X takes

the value r is proportional to 5^{-r}. Find $P(X < 3)$.

SECTION D

$[5 \times 4 = 20]$

(This section comprises of 4 long answer (LA) type questions of 5 marks each)

Q.32. Draw the rough sketch of the curve $y = 20\cos 2x; \left(\text{where} \frac{\pi}{6} \leq x \leq \frac{\pi}{3}\right)$.

U**sing integration,** find the area of the region bounded by the curve y = 20 cos2x from the ordinates $x = \frac{\pi}{6}$ to $x = \frac{\pi}{3}$ and the $x-$axis.

Q.33. The equation of the path traversed by the ball headed by the footballer is $y = ax^2 + bx + c; (where\ 0 \leq x \leq 14\ and\ a, b, c \in R\ and\ a \neq 0)$ with respect to a XY-coordinate system in the vertical plane. The ball passes through the points $(2,15)$, $(4,25)$ **and** $(14,15)$. Determine the values of a, b and c by solving the system of linear equations in a, b and c, using matrix method. Also find the equation of the path traversed by the ball.

Q.34. (a) If $f: R \to R$ is defined by $f(x) = |x|^3$, show that $f''(x)$ exists for all real x and find it.

OR

Q.34. (b) If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

Q.35. (a) Find the shortest distance between the lines l_1 and l_2 whose vector equations are

$$\vec{r} = \left(-\hat{\imath} - \hat{\jmath} - \hat{k}\right) + \lambda\left(7\hat{\imath} - 6\hat{\jmath} + \hat{k}\right) \text{ and } \vec{r} = \left(3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}\right) + \mu\left(\hat{\imath} - 2\hat{\jmath} + \hat{k}\right)$$

where λ and μ are parameters.

OR

Q.35. (b) Find the image of the point $(1,2,1)$ with respect to the line $\frac{x-3}{1} = \frac{y+1}{2} = \frac{z-1}{3}$. Also find the equation of the line joining the given point and its image.

SECTION- E

$[4 \times 3 = 12]$

(*This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each)*

Case Study-1

Q.36. Ramesh, the owner of a sweet selling shop, purchased some rectangular card board sheets of dimension **25*cm* by 40 *cm*** to make container packets without top. Let $x\,cm$ be the length of the side of the square to be cut out from each corner to give that sheet the shape of the container by folding up the flaps.

Based on the above information answer the following questions.

(i) Express the volume (V) of each container as function of x only. **[1 Mark]**

(ii) Find $\frac{dV}{dx}$ **[1 Mark]**

(iii) (a) For what value of x, the volume of each container is maximum? **[2 Marks]**

OR

(iii) (b) Check whether V has a point of inflection at x = $\frac{65}{6}$ or not? **[2 Marks]**

Case Study-2

Q.37. An organization conducted bike race under 2 different categories-boys and girls. In all, there were **250** participants. Among all of them finally three from Category **1** and two from Category **2** were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}, G = \{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race.

Ravi decides to explore these sets for various types of relations and functions.

On the basis of the above information, answer the following questions:

(i) Ravi wishes to form all the relations possible from B to G. How many such relations are possible? **[1 Mark]**

(ii) Write the smallest equivalence relation on **G**. **[1 Mark]**

(iii) (a) Ravi defines a relation from **B to B** as $R_1 = \{(b_1, b_2), (b_2, b_1)\}$. Write the minimum ordered pairs to be added in R_1 so that it becomes (A) reflexive but not symmetric, (B) reflexive and symmetric but not transitive. **[2 Marks]**

OR

(iii) (b) If the track of the final race (for the biker b_1) follows the curve $x^2 = 4y; (\text{where} 0 \le x \le 20\sqrt{2} \& 0 \le y \le 200)$, then state whether the track represents a one-one and onto function or not. (Justify). **[2 Marks]**

Case Study- 3

Q.38. Arka bought two cages of birds: Cage-I contains 5 parrots and 1 owl and Cage –II contains 6 parrots. One day Arka forgot to lock both cages and two birds flew from Cage-I to Cage-II (simultaneously). Then two birds flew back from cage-II to cage-I(simultaneously).

Assume that all the birds have equal chances of flying.

On the basis of the above information, answer the following questions:-

(i) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I then find the probability that the owl is still in Cage-I. **[2 *Marks*]**

(ii) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I, the owl is still seen in Cage-I, what is the probability that one parrot and the owl flew from Cage-I to Cage-II? **[2 *Marks*]**

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MARKING SCHEME

CLASS XII

MATHEMATICS (CODE-041)

SECTION: A (Solution of MCQs of 1 Mark each)

Q no.	ANS	HINTS/SOLUTION
1.	**(D)**	For a square matrix A of order $n\times n$, we have $A.(adj\,A)=|A|I_n$, where I_n is the identity matrix of order $n\times n$. So, $A.(adj\,A)=\begin{bmatrix}2025 & 0 & 0\\ 0 & 2025 & 0\\ 0 & 0 & 2025\end{bmatrix}=2025I_3 \Rightarrow |A|=2025$ & $|adj\,A|=|A|^{3-1}=(2025)^2$ $\therefore |A|+|adj\,A|=2025+(2025)^2$.
2.	**(A)**	P Order $p\times k$; Y Order $3\times k$ → For PY to exist $k=3$, Order of $PY=p\times k$ W Order $n\times 3$; Y Order $3\times k$ → Order of $WY=n\times k$ For $PY+WY$ to exist $\text{order}(PY)=\text{order}(WY)$ $\therefore p=n$
3.	**(C)**	$y=e^x \Rightarrow \frac{dy}{dx}=e^x$ In the domain (R) of the function, $\frac{dy}{dx}>0$, hence the function is strictly increasing in $(-\infty,\infty)$
4.	**(B)**	$|A|=5,\ |B^{-1}AB|^2=(|B^{-1}||A||B|)^2=|A|^2=5^2$.
5.	**(B)**	A differential equation of the form $\frac{dy}{dx}=f(x,y)$ is said to be homogeneous, if $f(x,y)$ is a homogeneous function of degree 0. Now, $x^n\frac{dy}{dx}=y\left(\log_e\frac{y}{x}+\log_e e\right)\Rightarrow\frac{dy}{dx}=\frac{y}{x^n}\left(\log_e e.\left(\frac{y}{x}\right)\right)=f(x,y);(Let)$. $f(x,y)$ will be a homogeneous function of degree 0, if $n=1$.
6.	**(A)**	**Method 1:** (***Short cut***) When the points $(x_1,y_1),(x_2,y_2)$ and (x_1+x_2,y_1+y_2) are collinear in the Cartesian plane then $\begin{vmatrix}x_1-x_2 & y_1-y_2\\ x_1-(x_1+x_2) & y_1-(y_1+y_2)\end{vmatrix}=0\Rightarrow\begin{vmatrix}x_1-x_2 & y_1-y_2\\ -x_2 & -y_2\end{vmatrix}=(-x_1y_2+x_2y_2+x_2y_1-x_2y_2)=0$ $\Rightarrow x_2y_1=x_1y_2$.

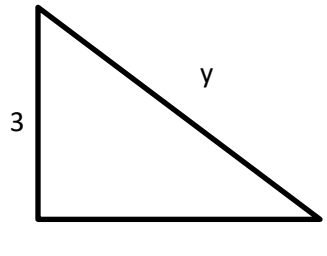
		Method 2: When the points $(x_1, y_1), (x_2, y_2)$ and $(x_1 + x_2, y_1 + y_2)$ are collinear in the Cartesian plane then $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_1 + x_2 & y_1 + y_2 & 1 \end{vmatrix} = 0$ $\Rightarrow 1.(x_2y_1 + x_2y_2 - x_1y_2 - x_2y_2) - 1(x_1y_1 + x_1y_2 - x_1y_1 - x_2y_1) + (x_1y_2 - x_2y_1) = 0$ $\Rightarrow x_2y_1 = x_1y_2$.
7.	**(A)**	$A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}$ When the matrix A is skew symmetric then $A^T = -A \Rightarrow a_{ij} = -a_{ji}$; $\Rightarrow c = -2; a = 0$ and $b = 3$ So , $a + b + c = 0 + 3 - 2 = 1$.
8.	**(C)**	$P(\overline{A}) = \frac{1}{2}; P(\overline{B}) = \frac{2}{3}; P(A \cap B) = \frac{1}{4}$ $\Rightarrow P(A) = \frac{1}{2}; P(B) = \frac{1}{3}$ We have, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{7}{12}$ $P\left(\frac{\overline{A}}{\overline{B}}\right) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{P(\overline{A \cup B})}{P(\overline{B})} = \frac{1 - P(A \cup B)}{P(\overline{B})} = \frac{1 - \frac{7}{12}}{\frac{2}{3}} = \frac{5}{8}$.
9.	**(B)**	For obtuse angle, $\cos\theta < 0 \Rightarrow \vec{p}.\vec{q} < 0$ $2\alpha^2 - 3\alpha + \alpha < 0 \Rightarrow 2\alpha^2 - 2\alpha < 0 \Rightarrow \alpha \in (0, 1)$
10.	**(C)**	$\lvert\vec{a}\rvert = 3, \lvert\vec{b}\rvert = 4, \lvert\vec{a} + \vec{b}\rvert = 5$ We have , $\lvert\vec{a} + \vec{b}\rvert^2 + \lvert\vec{a} - \vec{b}\rvert^2 = 2\left(\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2\right) = 2(9 + 16) = 50 \Rightarrow \lvert\vec{a} - \vec{b}\rvert = 5$.
11.	**(B)**	<table><tr><td>Corner point</td><td>Value of the objective function $Z = 4x + 3y$</td></tr><tr><td>1. $O(0,0)$</td><td>$z = 0$</td></tr><tr><td>2. $R(40,0)$</td><td>$z = 160$</td></tr><tr><td>3. $Q(30,20)$</td><td>$z = 120 + 60 = 180$</td></tr><tr><td>4. $P(0,40)$</td><td>$z = 120$</td></tr></table> Since , the feasible region is bounded so the maximum value of the objective function $z = 180$ is at $Q(30,20)$.

12.	**(A)**	$\int \frac{dx}{x^3(1+x^4)^{\frac{1}{2}}} = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{\frac{1}{2}}}$ (Let $1+x^{-4} = 1+\frac{1}{x^4} = t,\ dt = -4x^{-5}dx = -\frac{4}{x^5}dx \Rightarrow \frac{dx}{x^5} = -\frac{1}{4}dt$) $= -\frac{1}{4}\int \frac{dt}{t^{\frac{1}{2}}} = -\frac{1}{4} \times 2 \times \sqrt{t} + c$, where **'c'** denotes any arbitrary constant of integration. $= -\frac{1}{2}\sqrt{1+\frac{1}{x^4}} + c = -\frac{1}{2x^2}\sqrt{1+x^4} + c$
13.	**(A)**	We know, $\int_0^{2a} f(x)dx = 0,$ **if** $f(2a-x) = -f(x)$ Let $f(x) = \cos ec^7 x$. Now, $f(2\pi - x) = \cos ec^7(2\pi - x) = -\cos ec^7 x = -f(x)$ $\therefore \int_0^{2\pi} \cos ec^7 x\ dx = 0;$ Using the property $\int_0^{2a} f(x)dx = 0,$ **if** $f(2a-x) = -f(x).$
14.	**(B)**	The given differential equation $e^{y'} = x \Rightarrow \frac{dy}{dx} = \log x$ $dy = \log x\ dx \Rightarrow \int dy = \int \log x\ dx$ $y = x\log x - x + c$ hence the correct option is **(B)**.
15.	**(B)**	The graph represents $y = \cos^{-1} x$ whose domain is $[-1,1]$ and range is $[0,\pi]$.
16.	**(D)**	Since the inequality $Z = 18x + 10y < 134$ has no point in common with the feasible region hence the minimum value of the **objective function** $Z = 18x + 10y$ is **134** at $P(3,8)$.
17.	**(D)**	The graph of the function $f: R \to R$ defined by $f(x) = [x];$ $(\text{where}[.]\text{denotes } G.I.F)$ is a straight line $\forall x \in (2.5-h, 2.5+h)$, **'h'** is an infinitesimally small positive quantity. Hence, the function is continuous and differentiable at $x = 2.5$ **.**
18.	**(B)**	**The required region is symmetric about the** $y-$ axis. So, required area (in sq units) is $= \left\| 2\int_0^4 2\sqrt{y}dy \right\| = 4\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 = \frac{64}{3}.$
19.	**(A)**	**Both (A) and (R) are true and (R) is the correct explanation of (A).**
20.	**(A)**	**Both (A) and (R) are true and (R) is the correct explanation of (A).**

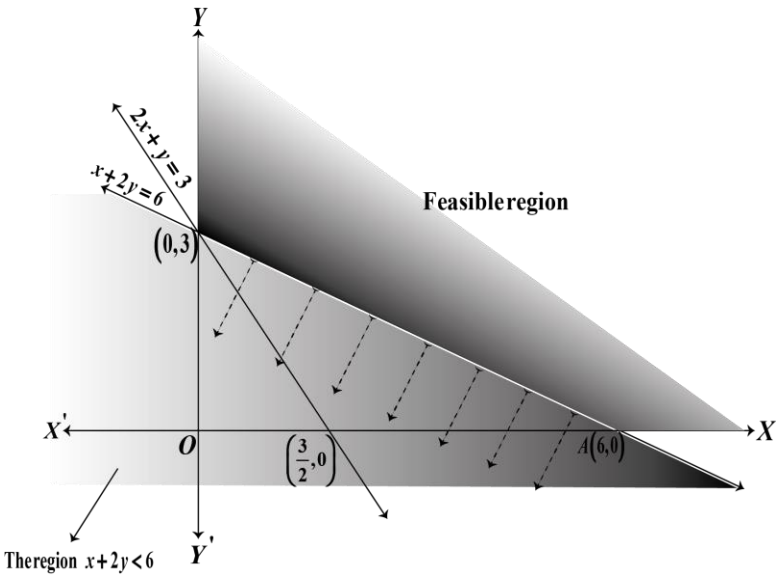
Section –B

[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21	$\cot^{-1}(3x+5) > \frac{\pi}{4} = \cot^{-1} 1$ =>3x + 5 < 1 (as $\cot^{-1}x$ is strictly decreasing function in its domain) => 3x < – 4 => $x < -\frac{4}{3}$ $\therefore x \in \left(-\infty, -\frac{4}{3}\right)$	**$\frac{1}{2}$** **$\frac{1}{2}$** **1**
22.	The marginal cost function is $\boldsymbol{C'(x) = 0.00039x^2 + 0.004x + 5}$. $\boldsymbol{C'(150) =}$ **₹ 14.375 .**	**1** **1**
23.(a)	$\boldsymbol{y = \tan^{-1} x}$ and $\boldsymbol{z = \log_e x}$. Then $\frac{dy}{dx} = \frac{1}{1+x^2}$ and $\frac{dz}{dx} = \frac{1}{x}$ $\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}}$ So, $= \frac{\frac{1}{1+x^2}}{\frac{1}{x}} = \frac{x}{1+x^2}$.	**$\frac{1}{2}$** **$\frac{1}{2}$** **$\frac{1}{2}$** **$\frac{1}{2}$**
OR **23.(b)**	Let $\boldsymbol{y = (\cos x)^x}$. Then, $\boldsymbol{y = e^{x \log_e \cos x}}$ On differentiating both sides with respect to $\boldsymbol{x}$, we get $\frac{dy}{dx} = e^{x \log_e \cos x} \frac{d}{dx}(x \log_e \cos x)$ $\Rightarrow \frac{dy}{dx} = (\cos x)^x \left\{ \log_e \cos x \frac{d}{dx}(x) + x \frac{d}{dx}(\log_e \cos x) \right\}$ $\Rightarrow \frac{dy}{dx} = (\cos x)^x \left\{ \log_e \cos x + x . \frac{1}{\cos x}(-\sin x) \right\} \Rightarrow \frac{dy}{dx} = (\cos x)^x (\log_e \cos x - x \tan x)$.	**$\frac{1}{2}$** **$\frac{1}{2}$** **1**
24.(a)	We have $\vec{b} + \lambda\vec{c} = (-1 + 3\lambda)\hat{i} + (2 + \lambda)\hat{j} + \hat{k}$ $(\vec{b} + \lambda\vec{c}) . \vec{a} = 0 \Rightarrow 2(-1 + 3\lambda) + 2(2 + \lambda) + 3 = 0$ $\lambda = -\frac{5}{8}$	**$\frac{1}{2}$** 1 **$\frac{1}{2}$**
OR **24.(b)**	$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = (4\hat{i} + 3\hat{k}) - \hat{k} = 4\hat{i} + 2\hat{k}$	**$\frac{1}{2}$**

	$\widehat{BA} = \frac{4}{2\sqrt{5}}\hat{\imath} + \frac{2}{2\sqrt{5}}\hat{k} = \frac{2}{\sqrt{5}}\hat{\imath} + \frac{1}{\sqrt{5}}\hat{k}$ So, the angles made by the vector $\overrightarrow{BA}$ with the ***x, y*** and the ***z*** axes are respectively $cos^{-1}\left(\frac{2}{\sqrt{5}}\right), \frac{\pi}{2}, cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$.	**$\frac{1}{2}$** **1**
25.	$\overrightarrow{d_1} = \vec{a} + \vec{b} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$, $\overrightarrow{d_2} = \vec{a} - \vec{b} = -6\hat{\jmath} - 8\hat{k}$ Area of the parallelogram = $\frac{1}{2}\left\|\overrightarrow{d_1} \times \overrightarrow{d_2}\right\| = \frac{1}{2}\left\|\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{vmatrix}\right\| = 2\left\|\hat{\imath} + 8\hat{\jmath} - 6\hat{k}\right\|$ Area of the parallelogram = $2\sqrt{101}$ sq. units.	**$\frac{1}{2}$** **1** **$\frac{1}{2}$**
	<u>Section –C</u> **[This section comprises of solution short answer type questions (SA) of 3 marks each]**	
26.	 $x^2 + 3^2 = y^2$ $When\ y = 5\ then\ x = 4, now\ 2x\ \frac{dx}{dt} = 2y\ \frac{dy}{dt}$ $4\ (200) = 5\ \frac{dy}{dt} \Rightarrow \frac{dy}{dt} = 160$ cm/s	**$\frac{1}{2}$** **$\frac{1}{2}$** **1** **1**
27.	$A = \frac{1}{3}\sqrt{t} \therefore \frac{dA}{dt} = \frac{1}{6}t^{-\frac{1}{2}} = \frac{1}{6\sqrt{t}}; \forall t \in (5,18)$ $\frac{dA}{dt} = \frac{1}{6\sqrt{t}} \therefore \frac{d^2A}{dt^2} = -\frac{1}{12t\sqrt{t}}$ So, $\frac{d^2A}{dt^2} < 0, \forall t \in (5,18)$ This means that the rate of change of the ability to understand spatial concepts decreases (slows down) with age.	**1** **1** **$\frac{1}{2}$** **$\frac{1}{2}$**
28(a)	**(i)** $\boldsymbol{\theta = \cos^{-1}\left(\frac{\vec{l_1}.\vec{l_2}}{\left\|\vec{l_1}\right\|.\left\|\vec{l_2}\right\|}\right) = \cos^{-1}\left(\frac{(\hat{\imath}-2\hat{\jmath}+3\hat{k}).(3\hat{\imath}-2\hat{\jmath}+\hat{k})}{\left\|(\hat{\imath}-2\hat{\jmath}+3\hat{k})\right\|\left\|(3\hat{\imath}-2\hat{\jmath}+\hat{k})\right\|}\right)}$ $= \boldsymbol{cos^{-1}\left(\frac{3+4+3}{\sqrt{1+4+9}\sqrt{9+4+1}}\right) = cos^{-1}\left(\frac{10}{14}\right) = cos^{-1}\left(\frac{5}{7}\right)}$. **(ii)** Scalar projection of $\vec{l_1}$ on $\vec{l_2}$ $= \frac{\vec{l_1}.\vec{l_2}}{\left\|\vec{l_2}\right\|} = \frac{(\hat{\imath}-2\hat{\jmath}+3\hat{k}).(3\hat{\imath}-2\hat{\jmath}+\hat{k})}{\left\|(3\hat{\imath}-2\hat{\jmath}+\hat{k})\right\|}$ $= \frac{3+4+3}{\sqrt{9+4+1}} = \frac{10}{\sqrt{14}}$.	1 **$\frac{1}{2}$** 1 **$\frac{1}{2}$**

28(b)	Line perpendicular to the lines $\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 7\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$. has a vector parallel it is given by $\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix} = 20\hat{i} + 10\hat{j} - 8\hat{k}$ $\therefore$ equation of line in vector form is $\vec{r} = -\hat{i} + 2\hat{j} + 7\hat{k} + a(10\hat{i} + 5\hat{j} - 4\hat{k})$ And equation of line in cartesian form is $\frac{x+1}{10} = \frac{y-2}{5} = \frac{z-7}{-4}$	1 1 1
29.(a)	$\int \left\{\frac{1}{log_e x} - \frac{1}{(log_e x)^2}\right\} dx$ $= \int \frac{dx}{log_e x} - \int \frac{1}{(log_e x)^2} dx = \frac{1}{log_e x}\int dx - \int \left\{\frac{d}{dx}\left(\frac{1}{log_e x}\right)\int dx\right\} dx - \int \frac{1}{(log_e x)^2} dx$ $= \frac{x}{log_e x} + \int \frac{1}{(log_e x)^2}\frac{1}{x}.x.dx - \int \frac{1}{(log_e x)^2} dx$ $= \frac{x}{log_e x} + \int \frac{1}{(log_e x)^2} dx - \int \frac{dx}{(log_e x)^2} = \frac{x}{log_e x} + c;$ where'c'is any arbitary constant of integration.	**1** **1** **1**
OR **29.(b)**	$\int_0^1 x(1-x)^n dx$ $= \int_0^1 (1-x)\{1-(1-x)\}^n dx, \left(as, \int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$ $= \int_0^1 x^n (1-x)dx$ $= \int_0^1 x^n dx - \int_0^1 x^{n+1} dx$ $= \frac{1}{n+1}[x^{n+1}]^1{}_0 - \frac{1}{n+2}[x^{n+2}]_0{}^1$ $= \frac{1}{n+1} - \frac{1}{n+2} \quad = \frac{1}{(n+1)(n+2)}.$	**1** **$\frac{1}{2}$** **$\frac{1}{2}$** **1**
30.	The feasible region determined by the constraints, $\mathbf{2x + y \geq 3,\ x + 2y \geq 6,\ x \geq 0, y \geq 0}$ is as shown.	



The corner points of the **unbounded** feasible region are $A(6,0)$ and $B(0,3)$.

The values of Z at these corner points are as follows:

Corner point	Value of the objective function $Z = x + 2y$
$A(6,0)$	6
$B(0,3)$	6

We observe the region $x + 2y < 6$ have no points in common with the unbounded feasible region. Hence the minimum value of $z = 6$.

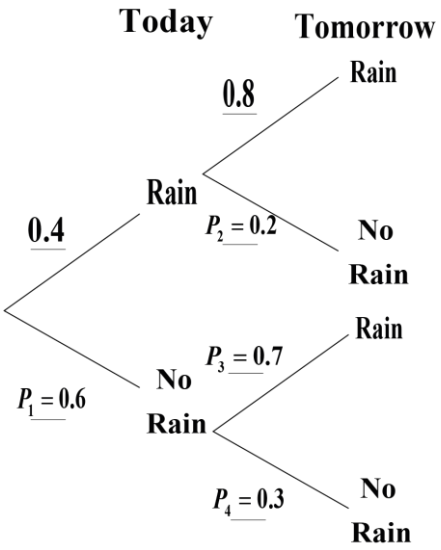
It can be seen that the value of Z at points **A** and **B** is same. If we take any other point on the line $x + 2y = 6$ such as (2,2) on line $x + 2y = 6$, then $Z = 6$.

Thus, the minimum value of Z occurs for more than **2** points, and is equal to 6.

31.(a) Since the event of raining today and not raining today are complementary events so if the probability that it rains today is 0.4 then the probability that it does not rain today is $1 - 0.4 = 0.6 \Rightarrow P_1 = 0.6$

If it rains today, the probability that it will rain tomorrow is 0.8 then the probability that it will not rain tomorrow is $\mathbf{1-0.8=0.2}$.

If it does not rain today, the probability that it will rain tomorrow is 0.7 then the probability that it will not rain tomorrow is $\mathbf{1-0.7=0.3}$



(i) $\boldsymbol{P_1 \times P_4 - P_2 \times P_3 = 0.6 \times 0.3 - 0.2 \times 0.7 = 0.04.}$ **1**

(ii) Let $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$ be the events that it will rain today and it will not rain today respectively. **1**

$\boldsymbol{P(E_1) = 0.4}$ **&** $\boldsymbol{P(E_2) = 0.6}$

$\boldsymbol{A}$ be the event that it will rain tomorrow. $P\left(\frac{A}{E_1}\right) = 0.8$ & $P\left(\frac{A}{E_2}\right) = 0.7$ **$\frac{1}{2}$**

We have, $P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) = 0.4 \times 0.8 + 0.6 \times 0.7 = 0.74.$

The probability of rain tomorrow is **0.74.** **$\frac{1}{2}$**

OR

31.(b)

Given $P(X = r)\alpha\frac{1}{5^r}$ **$\frac{1}{2}$**

$P(X = r) = k\frac{1}{5^r}$, (where $\boldsymbol{k}$ is a non-zero constant)

$P(r = 0) = k.\frac{1}{5^0}$

$P(r = 1) = k.\frac{1}{5^1}$

$P(r = 2) = k.\frac{1}{5^2}$ **$\frac{1}{2}$**

$P(r = 3) = k.\frac{1}{5^3}$

…….

…….

We have, $P(X = 0) + P(X = 1) + P(X = 2) + \ldots\ldots\ldots\ldots = 1$ **$\frac{1}{2}$**

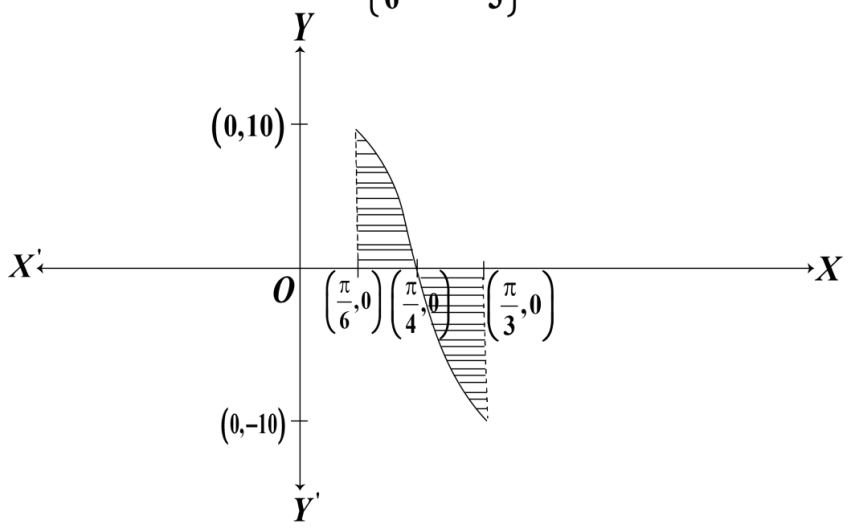
	$\Rightarrow k\left(1+\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\ldots\ldots\ldots\ldots\right)=1$ $\Rightarrow k\left(\frac{1}{1-\frac{1}{5}}\right)=1 \Rightarrow k=\frac{4}{5}$ So, $P(X<3)=P(X=0)+P(X=1)+P(X=2)$ $=\frac{4}{5}\left(1+\frac{1}{5}+\frac{1}{5^2}\right)=\frac{4}{5}\left(\frac{25+5+1}{25}\right)=\frac{124}{125}$.	**$\frac{1}{2}$** **1**

Section –D

[This section comprises of solution of long answer type questions (LA) of 5 marks each]

32.

$y = 20\cos 2x\ ;\ \left\{\frac{\pi}{6} \le x \le \frac{\pi}{3}\right\}$



1

Required area $= 20\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} cos2x\ dx + \left|20\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} cos2x\ dx\right|$ **1+1**

$= 20\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} + \left|20\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}\right|$ **1**

$= 10\left(1-\frac{\sqrt{3}}{2}\right) + 10\left(1-\frac{\sqrt{3}}{2}\right) = 20\left(1-\frac{\sqrt{3}}{2}\right)$ sq. units. **1**

33.

$\boldsymbol{y = ax^2 + bx + c}$

$\boldsymbol{15 = 4a + 2b + c}$ **1**

$\boldsymbol{25 = 16a + 4b + c}$

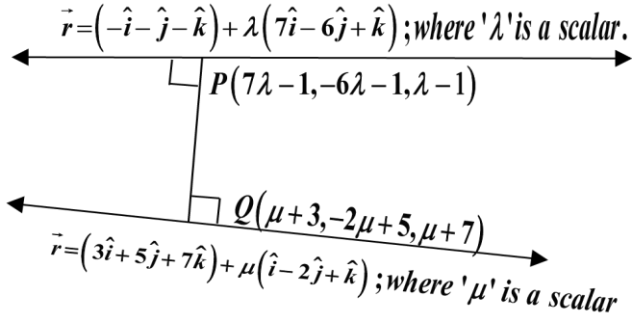
$\boldsymbol{15 = 196a + 14b + c}$

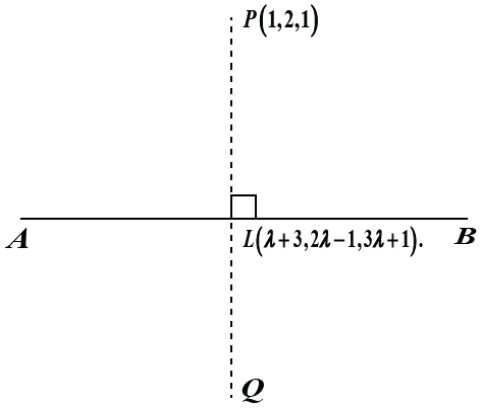
The set of equations can be represented in the matrix form as $\boldsymbol{AX = B}$, **$\frac{1}{2}$**

where $A=\begin{bmatrix}4 & 2 & 1\\ 16 & 4 & 1\\ 196 & 14 & 1\end{bmatrix}$, $X=\begin{bmatrix}a\\ b\\ c\end{bmatrix}$ and $B=\begin{bmatrix}15\\ 25\\ 15\end{bmatrix} \Rightarrow \begin{bmatrix}4 & 2 & 1\\ 16 & 4 & 1\\ 196 & 14 & 1\end{bmatrix}\begin{bmatrix}a\\ b\\ c\end{bmatrix}=\begin{bmatrix}15\\ 25\\ 15\end{bmatrix}$.

$|A| = 4(4-14) - 2(16-196) + (224-784) = -40 + 360 - 560 = -240 \neq 0$. Hence $\boldsymbol{A^{-1}}$ exists. **$\frac{1}{2}$**

	Now, $adj(A) = \begin{bmatrix} -10 & 180 & -560 \\ 12 & -192 & 336 \\ -2 & 12 & -16 \end{bmatrix}^T = \begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}$	**1**
	$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = -\frac{1}{240}\begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}\begin{bmatrix} 15 \\ 25 \\ 15 \end{bmatrix} = -\frac{5}{240}\begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix} = -\frac{5}{240}\begin{bmatrix} 24 \\ -384 \\ -48 \end{bmatrix}$	**1**
	$\therefore \boldsymbol{a = -\frac{1}{2}, b = 8, c = 1}$	**$\frac{1}{2}$**
	So, the equation becomes $\boldsymbol{y = -\frac{1}{2}x^2 + 8x + 1}$	**$\frac{1}{2}$**
34.(a)	We have, $f(x) = \|x\|^3, \begin{cases} x^3, \text{if } x \geq 0 \\ (-x)^3 = -x^3, \text{if} x < 0 \end{cases}$	**$\frac{1}{2}$**
	Now, $(LHD \text{ at } x = 0) = \lim_{x \to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x \to 0^-} \left(\frac{-x^3-0}{x}\right) = \lim_{x \to 0^-} (-x^2) = 0$	**$\frac{1}{2}$**
	$(RHD \text{ at} x = 0) \lim_{x \to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x \to 0^+} \left(\frac{x^3-0}{x}\right) = \lim_{x \to 0} (-x^2) = 0$	**$\frac{1}{2}$**
	$\therefore (LHD \text{ of } f(x) \text{ at } x = 0) = (RHD \text{ of } f(x) \text{ at } x = 0)$	**$\frac{1}{2}$**
	So, $\boldsymbol{f(x)}$ is differentiable at $\boldsymbol{x = 0}$ and the derivative of $\boldsymbol{f(x)}$ is given by	
	$f'(x) = \begin{cases} 3x^2, \text{if} x \geq 0 \\ -3x^2, \text{if} x < 0 \end{cases}$	**1**
	Now, $(LHD off'(x) at x = 0) = \lim_{x \to 0^-} \frac{f'(x)-f'(0)}{x-0} = \lim_{x \to 0^-} \left(\frac{-3x^2-0}{x}\right) = \lim_{x \to 0^-} (-3x) = 0$	**$\frac{1}{2}$**
	$(RHD \text{ off}'(x) \text{ at } x = 0) = \lim_{x \to 0^+} \frac{f'(x)-f'(0)}{x-0} = \lim_{x \to 0^+} \left(\frac{3x^2-0}{x-0}\right) = \lim_{x \to 0^+} (3x) = 0$	**$\frac{1}{2}$**
	$\therefore (LHD \text{ off}'(x) at x = 0) = (RHD off'(x) at x = 0)$	**$\frac{1}{2}$**
	So, $f'(x)$is differentiable at $\boldsymbol{x = 0.}$	**$\frac{1}{2}$**
	Hence, $f''(x) = \begin{cases} 6x, \text{if} x \geq 0 \\ -6x, \text{if} x < 0. \end{cases}$	**$\frac{1}{2}$**
OR **34 .(b)**	Given relation is $(x-a)^2 + (y-b)^2 = c^2, c > 0.$	
	Let $\boldsymbol{x - a = c\cos\theta}$ and $y - b = c \sin\theta$.	**$\frac{1}{2}$**
	Therefore, $\frac{dx}{d\theta} = -c \sin\theta$ And $\frac{dy}{d\theta} = c \cos\theta$	**$\frac{1}{2}$**
	$\therefore \frac{dy}{dx} = -\cot\theta$	**1**
	Differentiate both sides with respect to θ, we get $\frac{d}{d\theta}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}(-\cot\theta)$	**$\frac{1}{2}$**

	Or, $\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{d\theta} = cos\,ec^2\theta$ Or, $\frac{d^2y}{dx^2}(-c\,sin\,\theta) = \text{cosec}^2\theta$ $\frac{d^2y}{dx^2} = -\frac{cosec^3\theta}{c}$ $\therefore \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{c[1+cot^2\,\theta]^{\frac{3}{2}}}{-cos\,ec^3\theta} = \frac{-c(cos\,ec^2\theta)^{\frac{3}{2}}}{\text{cosec}^3\theta} = -c,$ Which is constant and is independent of $\boldsymbol{a}$ and $\boldsymbol{b}$.	$\frac{1}{2}$ $\frac{1}{2}$ **1** $\frac{1}{2}$
35.(a)	 Given that equation of lines are $\vec{r} = \left(-\hat{\imath}-\hat{\jmath}-\hat{k}\right)+\lambda\left(7\hat{\imath}-6\hat{\jmath}+\hat{k}\right)\ldots\ldots\ldots\ldots\ldots(i)$ and $\vec{r} = \left(3\hat{\imath}+5\hat{\jmath}+7\hat{k}\right)+\mu\left(\hat{\imath}-2\hat{\jmath}+\hat{k}\right)\ldots\ldots\ldots\ldots\ldots(ii)$ The given lines are non-parallel lines as vectors $7\hat{\imath}-6\hat{\jmath}+\hat{k}$ and $\hat{\imath}-2\hat{\jmath}+\hat{k}$ are not parallel. There is a unique line segment $\boldsymbol{PQ}$ ($\boldsymbol{P}$ lying on line $(\boldsymbol{i})$ and $\boldsymbol{Q}$ on the other line $(\boldsymbol{ii})$), which is at right angles to both the lines $\boldsymbol{PQ}$ is the shortest distance between the lines. Hence, the shortest possible distance between the lines $= \boldsymbol{PQ}$. Let the position vector of the point $\boldsymbol{P}$ lying on the line $\vec{r} = \left(-\hat{\imath}-\hat{\jmath}-\hat{k}\right)+\lambda\left(7\hat{\imath}-6\hat{\jmath}+\hat{k}\right)$ where '$\boldsymbol{\lambda}$' is a scalar, is $(7\lambda-1)\hat{\imath}-(6\lambda+1)\hat{\jmath}+(\lambda-1)\hat{k}$, for some $\boldsymbol{\lambda}$ and the position vector of the point $\boldsymbol{Q}$ lying on the line $\vec{r} = \left(3\hat{\imath}+5\hat{\jmath}+7\hat{k}\right)+\mu\left(\hat{\imath}-2\hat{\jmath}+\hat{k}\right)$ where '$\boldsymbol{\mu}$' is a scalar, is $(\mu+3)\hat{\imath}+(-2\mu+5)\hat{\jmath}+(\mu+7)\hat{k}$, for some $\boldsymbol{\mu}$. Now, the vector $\overrightarrow{PQ} = \overrightarrow{OQ}-\overrightarrow{OP} = (\mu+3-7\lambda+1)\hat{\imath}+(-2\mu+5+6\lambda+1)\hat{\jmath}+(\mu+7-\lambda+1)\hat{k}$ $i.e., \overrightarrow{PQ} = (\mu-7\lambda+4)\hat{\imath}+(-2\mu+6\lambda+6)\hat{\jmath}+(\mu-\lambda+8)\hat{k}$; (where '$\boldsymbol{O}$' is the origin), is perpendicular to both the lines, so the vector $\overrightarrow{PQ}$ is perpendicular to both the vectors $7\hat{\imath}-6\hat{\jmath}+\hat{k}$ and $\hat{\imath}-2\hat{\jmath}+\hat{k}$. $\Rightarrow(\mu-7\lambda+4).7+(-2\mu+6\lambda+6).(-6)+(\mu-\lambda+8).1=0$	$\frac{1}{2}$ $\frac{1}{2}$ **1**

	$\&(\mu - 7\lambda + 4).1 + (-2\mu + 6\lambda + 6).(-2) + (\mu - \lambda + 8).1 = 0$	
	$\Rightarrow \mathbf{20\mu - 86\lambda = 0} \Rightarrow \mathbf{10\mu - 43\lambda = 0} \& 6\mu - 20\lambda = 0 \Rightarrow 3\mu - 10\lambda = 0$	**1**
	On solving the above equations, we get $\boldsymbol{\mu = \lambda = 0}$	$\frac{\mathbf{1}}{\mathbf{2}}$
	So, the position vector of the points $\boldsymbol{P}$ and $\boldsymbol{Q}$ are $-\hat{\imath} - \hat{\jmath} - \hat{k}$ and $3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}$ respectively.	$\frac{\mathbf{1}}{\mathbf{2}}$
	$\overrightarrow{PQ} = 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$ and	
	$\left\|\overrightarrow{PQ}\right\| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116} = 2\sqrt{29}\ units.$	**1**
OR **35.(b)**		
	Let $\boldsymbol{P(1, 2, 1)}$ be the given point and $\boldsymbol{L}$ be the foot of the perpendicular from $\boldsymbol{P}$ to the given line $\boldsymbol{AB}$ $(as\ shown\ in\ the\ figure\ above)$.	
	Let's put $\frac{\boldsymbol{x-3}}{\mathbf{1}} = \frac{\boldsymbol{y+1}}{\mathbf{2}} = \frac{\boldsymbol{z-1}}{\mathbf{3}} = \boldsymbol{\lambda}$. Then, $\boldsymbol{x = \lambda + 3, y = 2\lambda - 1, z = 3\lambda + 1}$	$\frac{\mathbf{1}}{\mathbf{2}}$
	Let the coordinates of the point $\boldsymbol{L}$ be $\boldsymbol{(\lambda + 3, 2\lambda - 1, 3\lambda + 1)}$.	
	So, direction ratios of $\boldsymbol{PL}$ are $(\lambda + 3 - 1, 2\lambda - 1 - 2, 3\lambda + 1 - 1) i.e., (\lambda + 2, 2\lambda - 3, 3\lambda)$	
	Direction ratios of the given line are **1, 2 and 3,** which is perpendicular to $\boldsymbol{PL}$. Therefore, we have,	$\frac{\mathbf{1}}{\mathbf{2}}$
	$(\lambda + 2).1 + (2\lambda - 3).2 + 3\lambda.3 = 0 \Rightarrow 14\lambda = 4 \Rightarrow \lambda = \frac{2}{7}$	$\frac{\mathbf{1}}{\mathbf{2}}$
	Then, $\lambda + 3 = \frac{2}{7} + 3 = \frac{23}{7}$; $2\lambda - 1 = 2\left(\frac{2}{7}\right) - 1 = -\frac{3}{7}$; $3\lambda + 1 = 3\left(\frac{2}{7}\right) + 1 = \frac{13}{7}$	$\frac{\mathbf{1}}{\mathbf{2}}$
	Therefore, coordinates of the point $\boldsymbol{L}$ are $\left(\frac{23}{7}, -\frac{3}{7}, \frac{13}{7}\right)$.	
	Let $Q(x_1, y_1, z_1)$ be the image of $\boldsymbol{P(1, 2, 1)}$ with respect to the given line. Then, $\boldsymbol{L}$ is the mid-point of $\boldsymbol{PQ}$.	**1**
	Therefore, $\frac{1+x_1}{2} = \frac{23}{7}, \frac{2+y_1}{2} = -\frac{3}{7}, \frac{1+z_1}{2} = \frac{13}{7} \Rightarrow x_1 = \frac{39}{7}, y_1 = -\frac{20}{7}, z_1 = \frac{19}{7}$	
	Hence, the image of the point $\boldsymbol{P(1,2,1)}$ with respect to the given line $Q\left(\frac{39}{7}, -\frac{20}{7}, \frac{19}{7}\right)$.	**1**
	The equation of the line joining $\boldsymbol{P(1,2,1)}$ and $Q\left(\frac{39}{7}, -\frac{20}{7}, \frac{19}{7}\right)$ is	

	$\frac{x-1}{32/7}=\frac{y-2}{-34/7}=\frac{z-1}{12/7}\Rightarrow\frac{x-1}{16}=\frac{y-2}{-17}=\frac{z-1}{6}$.	**1**

Section –E

[This section comprises solution of 3 case- study/passage based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)

36.	(i) $V=(40-2x)(25-2x)xcm^3$	**1**
	(ii) $\frac{dV}{dx}=4(3x-50)(x-5)$	**1**
	(iii) (a) For extreme values $\frac{dV}{dx}=4(3x-50)(x-5)=0$	**$\frac{1}{2}$**
	$\Rightarrow x=\frac{50}{3}$ or $x=5$	**$\frac{1}{2}$**
	$\frac{d^2V}{dx^2}=24x-260$	**$\frac{1}{2}$**
	$\therefore \frac{d^2V}{dx^2}$ at $x=5$ is $-140<0$	**$\frac{1}{2}$**
	$\therefore V$ is max *when* $x=5$	
	(iii) **OR**	
	(b) For extreme values $\frac{dV}{dx}=4(3x^2-65x+250)$	**$\frac{1}{2}$**
	$\frac{d^2V}{dx^2}=4(6x-65)$	**$\frac{1}{2}$**
	$\frac{dV}{dx}$ *at* $x=\frac{65}{6}$ exists and $\frac{d^2V}{dx^2}$ *at* $x=\frac{65}{6}$ *is* 0.	
	$\frac{d^2V}{dx^2}$ *at* $x=\left(\frac{65}{6}\right)^-$ is negative and $\frac{d^2V}{dx^2}$ *at* $x=\left(\frac{65}{6}\right)^+$ is positive	**$\frac{1}{2}$**
	$\therefore x=\frac{65}{6}$ is a point of inflection.	**$\frac{1}{2}$**
37.	**(i) Number of relations is equal to the number of subsets of the set $B\times G=2^{n(B\times G)}$ $=2^{n(B)\times n(G)}=2^{3\times2}=2^6$**	**1**
	($Wheren(A)$ denotes the number of the elements in the finite set A)	
	(ii) Smallest Equivalence relation on G is $\{(g_1,g_1),(g_2,g_2)\}$	**1**
	(iii) (a) (A) reflexive but not symmetric = $\{(b_1,b_2),(b_2,b_1),(b_1,b_1),(b_2,b_2),(b_3,b_3),(b_2,b_3)\}$.	

	So the minimum number of elements to be added are $(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3)$ {Note : it can be any one of the pair from, (b_3, b_2), (b_1, b_3), (b_3, b_1) **in place of** (b_2, b_3) $\boldsymbol{also}$**}** (B) reflexive and symmetric but not transitive = $\{(b_1, b_2), (b_2, b_1), (b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)\}$.	**1**
	So the minimum number of elements to be added are $(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)$	**1**
	OR (iii) (b) One-one and onto function $x^2 = 4y$. let $y = f(x) = \frac{x^2}{4}$ Let $x_1, x_2 \in [0, 20\sqrt{2}]$ such that $f(x_1) = f(x_2) \Rightarrow \frac{{x_1}^2}{4} = \frac{{x_1}^2}{4}$ $\Rightarrow {x_1}^2 = {x_2}^2 \Rightarrow (x_1 - x_2)(x_1 + x_2) = 0 \Rightarrow x_1 = x_2$ as $x_1, x_2 \in [0, 20\sqrt{2}]$ $\therefore f$ is one-one function Now, $0 \leq y \leq 200$ hence the value of y is non-negative and $f(2\sqrt{y}) = y$ $\therefore$ for any arbitrary $y \in [0, 200]$, the pre-image of y exists in $[0, 20\sqrt{2}]$ hence f is onto function.	**1** **1**
38.	Let E_1 be the event that one parrot and one owl flew from cage –I E_2 be the event that two parrots flew from Cage-I A be the event that the owl is still in cage-I (i) Total ways for A to happen From cage I 1 parrot and 1 owl flew and then from Cage-II 1 parrot and 1 owl flew back + From cage I 1 parrot and 1 owl flew and then from Cage-II 2 parrots flew back + From cage I 2 parrots flew and then from Cage-II 2 parrots came back. $=(5_{C_1} \times 1_{C_1})(7_{C_1} \times 1_{C_1}) + (5_{C_1} \times 1_{C_1})(7_{C_2}) + (5_{C_2})(8_{C_2})$ Probability that the owl is still in cage –I = P($E_1 \cap A$) + P($E_2 \cap A$) $\frac{(5_{C_1} \times 1_{C_1})(7_{C_1} \times 1_{C_1}) + (5_{C_2})(8_{C_2})}{(5_{C_1} \times 1_{C_1})(7_{C_1} \times 1_{C_1}) + (5_{C_1} \times 1_{C_1})(7_{C_2}) + (5_{C_2})(8_{C_2})}$ $= \frac{35 + 280}{35 + 105 + 280} = \frac{315}{420} = \frac{3}{4}$	**$\frac{1}{2}$** **1** **$\frac{1}{2}$**

	(i) The probability that one parrot and the owl flew from Cage-I to Cage-II given that the owl is still in cage-I is $P\left(E_1/_A\right)$ $P\left(E_1/_A\right) = \frac{P(E_1 \cap A)}{P(E_1 \cap A)+P(E_2 \cap A)}$ (by Baye's Theorem) $= \frac{\frac{35}{420}}{\frac{315}{420}} = \frac{1}{9}$	**$\frac{1}{2}$** **$\frac{1}{2}$** **1**