SET-1

Series : PQSR5

प्रश्न-पत्र कोड
Q.P. Code **65/5/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

निर्धारित समय : ***3*** *घण्टे* | *अधिकतम अंक :* ***80***
Time allowed : ***3*** *hours* | *Maximum Marks :* ***80***

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **23** printed pages.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **38** questions.
- **Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. [ ]

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सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में **38** प्रश्न हैं। **सभी** प्रश्न **अनिवार्य** हैं।*

(ii) *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ**।*

(iii) ***खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **1** अंक के प्रश्न हैं।*

(iv) ***खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **2** अंकों के प्रश्न हैं।*

(v) ***खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **3** अंकों के प्रश्न हैं।*

(vi) ***खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **5** अंकों के प्रश्न हैं।*

(vii) ***खण्ड ङ** में प्रश्न संख्या **36** से **38** तक प्रकरण अध्ययन आधारित **4** अंकों के प्रश्न हैं।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

1. यदि $A = \begin{bmatrix} -p & q \\ r & p \end{bmatrix}$ ऐसा आव्यूह है कि $A^2 = I$ है, तो :

(A) $1 + p^2 + qr = 0$ (B) $1 - p^2 - qr = 0$

(C) $1 - p^2 + qr = 0$ (D) $1 + p^2 - qr = 0$

2. यदि A एक ऐसा वर्ग आव्यूह है कि $A^2 = A$ है, तो $(A - I)^3 - A$ बराबर है :

(A) I (B) – I

(C) A (D) A^2

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General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A**, **B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises Multiple Choice Questions (MCQs) of 1 mark each.

1. If matrix $A = \begin{bmatrix} -p & q \\ r & p \end{bmatrix}$ is such that $A^2 = I$, then :

(A) $1 + p^2 + qr = 0$ (B) $1 - p^2 - qr = 0$

(C) $1 - p^2 + qr = 0$ (D) $1 + p^2 - qr = 0$

2. If A is a square matrix such that $A^2 = A$, then $(A - I)^3 - A$ is equal to :

(A) I (B) – I

(C) A (D) A^2

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3. प्रतिलोम त्रिकोणमितीय फलनों के लिए, निम्नलिखित में से कौन-सी मुख्य मानीय शाखा सही परिभाषित ***नहीं*** है ?

(A) $\tan^{-1} : R \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(B) $\sec^{-1} : R - (-1, 1) \to [0, \pi] - \left\{\frac{\pi}{2}\right\}$

(C) $\cot^{-1} : R \to (0, \pi)$

(D) $\text{cosec}^{-1} : R - (-1, 1) \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

4. माना $A = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ तथा $B = \begin{bmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix}$ है । यदि $A + B + C = O$ है, तो आव्यूह C है :

(A) $\begin{bmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 3 & 5 \\ -3 & -4 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} -3 & -3 & -5 \\ 3 & 4 & 2 \end{bmatrix}$

5. यदि A एक व्युत्क्रमणीय आव्यूह है, तो निम्नलिखित में से कौन-सा सत्य ***नहीं*** है ?

(A) adj A अव्युत्क्रमणीय है (B) $(\text{adj } A)^{-1} = (\text{adj } A^{-1})$

(C) $|A| \neq 0$ (D) A^{-1} का अस्तित्व है

6. यदि $f(x) = \begin{cases} \frac{x^2 - 4x - 5}{x + 1}, & x \neq -1 \\ k, & x = -1 \end{cases}$

$x = -1$ पर एक संतत फलन है, तो k का मान है :

(A) कोई वास्तविक मान (B) 6

(C) -1 (D) -6

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3. For the inverse trigonometric functions, which of the following Principal Value Branch is ***not*** correctly defined ?

(A) $\tan^{-1} : R \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(B) $\sec^{-1} : R - (-1, 1) \to [0, \pi] - \left\{\frac{\pi}{2}\right\}$

(C) $\cot^{-1} : R \to (0, \pi)$

(D) $\text{cosec}^{-1} : R - (-1, 1) \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

4. Let $A = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix}$. If $A + B + C = O$, then matrix C is :

(A) $\begin{bmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 3 & 5 \\ -3 & -4 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} -3 & -3 & -5 \\ 3 & 4 & 2 \end{bmatrix}$

5. If A is a non-singular matrix, then which of the following is ***not*** true ?

(A) adj A is singular (B) $(\text{adj } A)^{-1} = (\text{adj } A^{-1})$

(C) $|A| \neq 0$ (D) A^{-1} exists

6. If $f(x) = \begin{cases} \frac{x^2 - 4x - 5}{x + 1}, & x \neq -1 \\ k, & x = -1 \end{cases}$

is continuous at $x = -1$, then the value of k is :

(A) Any real value (B) 6

(C) -1 (D) -6

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7. यदि Δ ABC जिसके शीर्ष A(3, 1), B(– 2, 1) तथा C(0, k) हैं, का क्षेत्रफल 5 वर्ग इकाई है, तो k के मान हैं :

(A) 3, 1 (B) –1, 3

(C) –1, 2 (D) 0, 2

8. $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right), -\frac{\pi}{4} < x < \frac{\pi}{4}$ का x के सापेक्ष अवकलज है :

(A) –1 (B) 1

(C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

9. फलन $f(x) = (x - 2)^2 + 5$ का अंतराल [– 3, 2] में निरपेक्ष निम्नतम मान है :

(A) – 3 (B) 2

(C) 5 (D) 30

10. $\int \frac{1}{\sqrt{1 + \cos 2x}} dx$ बराबर है :

(A) $\log \cos x + C$ (B) $\frac{1}{\sqrt{2}} \log |\sec x + \tan x| + C$

(C) $\frac{1}{\sqrt{2}} \log |\sec x - \tan x| + C$ (D) $\log \sin 2x + C$

11. $\int_{-5}^{-1} \frac{1}{x} dx$ का मान है :

(A) $-\log 5$ (B) x^6

(C) $\log (-5)$ (D) x^{-6}

12. एक चींटी को काग़ज़ की एक शीट पर एक सीधी रेखा में रेंगते हुए देखा गया है जिसका समीकरण $y = 2x - 4$ द्वारा प्रदत्त है। चींटी द्वारा तय किए गए पृष्ठ का क्षेत्रफल जो y-अक्ष, x-अक्ष तथा $x = 1$ द्वारा परिबद्ध है, है :

(A) 1 वर्ग इकाई (B) 3 वर्ग इकाई

(C) 2 वर्ग इकाई (D) 4 वर्ग इकाई

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7. If the area of Δ ABC with vertices A(3, 1), B(– 2, 1) and C(0, k) is 5 sq. units, then values of k are :

(A) 3, 1 (B) $-1, 3$

(C) $-1, 2$ (D) 0, 2

8. Derivative of $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$, $-\frac{\pi}{4} < x < \frac{\pi}{4}$ with respect to x is :

(A) -1 (B) 1

(C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

9. Absolute minimum value of $f(x) = (x-2)^2 + 5$ in the interval $[-3, 2]$ is :

(A) – 3 (B) 2

(C) 5 (D) 30

10. $\int \frac{1}{\sqrt{1+\cos 2x}}\,dx$ is equal to :

(A) $\log \cos x + C$ (B) $\frac{1}{\sqrt{2}} \log |\sec x + \tan x| + C$

(C) $\frac{1}{\sqrt{2}} \log |\sec x - \tan x| + C$ (D) $\log \sin 2x + C$

11. The value of $\int_{-5}^{-1} \frac{1}{x}\,dx$ is equal to :

(A) $-\log 5$ (B) x^6

(C) $\log(-5)$ (D) x^{-6}

12. An ant is observed crawling on a sheet of paper along a straight line given by equation $y = 2x - 4$. Area of the surface covered by the ant bounded by y-axis, x-axis and $x = 1$ is :

(A) 1 sq. unit (B) 3 sq. units

(C) 2 sq. units (D) 4 sq. units

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13. अवकल समीकरण $1 + \left(\frac{d^3y}{dx^3}\right)^3 = \lambda \frac{d^2y}{dx^2}$ की कोटि तथा घात है :

(A) कोटि = 3, घात = 3 (B) कोटि = 2, घात = 2

(C) कोटि = 3, घात = 1 (D) कोटि = 2, घात = 1

14. अवकल समीकरण $\frac{dy}{dx} = e^{3x-y}$ का व्यापक हल है :

(A) $3e^y = e^{3x} + C$ (B) $\log(3x - y) = C$

(C) $e^{3x-y} = C$ (D) $-e^y + 3e^{3x} = C$

15. रैखिक व्यवरोधों के एक निकाय द्वारा बने सुसंगत क्षेत्र के कोनीय बिंदु (0, 0), (0, 40), (20, 40), (60, 20) तथा (60, 0) हैं। यदि LPP का उद्देश्य फलन $Z = 4x + 3y$ है, तो इसका अधिकतम मान है :

(A) 200 (B) 300

(C) 240 (D) 120

16. यदि बिंदु (24, n) का स्थिति सदिश $\vec{p}$ इस प्रकार है कि $|\vec{p}| = 25$ है, तो n के मान हैं :

(A) ± 49 (B) ± 5

(C) ± 1 (D) ± 7

17. यदि सदिश $\vec{a} = 3\hat{i} + 2\hat{j} + \lambda\hat{k}$ तथा $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$, एक डॉक्टर के क्लिनिक के बाहर लगे हुए रेड क्रॉस साइन की दो स्ट्रिप्स का निरूपण करते हैं, तो λ का मान है :

(A) 1 (B) $\frac{5}{2}$

(C) $\frac{2}{5}$ (D) 0

13. The order and degree of the differential equation

$1 + \left(\frac{d^3y}{dx^3}\right)^3 = \lambda \frac{d^2y}{dx^2}$ is :

(A) Order = 3, Degree = 3 (B) Order = 2, Degree = 2

(C) Order = 3, Degree = 1 (D) Order = 2, Degree = 1

14. The general solution for the differential equation $\frac{dy}{dx} = e^{3x-y}$ is :

(A) $3e^y = e^{3x} + C$ (B) $\log(3x - y) = C$

(C) $e^{3x-y} = C$ (D) $-e^y + 3e^{3x} = C$

15. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40) (60, 20) and (60, 0). If the objective function of an LPP is Z = 4x + 3y, then the maximum value is :

(A) 200 (B) 300

(C) 240 (D) 120

16. If position vector $\vec{p}$ of a point (24, n) is such that $|\vec{p}| = 25$, then the value of n is :

(A) ± 49 (B) ± 5

(C) ± 1 (D) ± 7

17. If vectors $\vec{a} = 3\hat{i} + 2\hat{j} + \lambda\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$, represent the two strips of the Red Cross sign placed outside a doctor's clinic, then the value of λ is :

(A) 1 (B) $\frac{5}{2}$

(C) $\frac{2}{5}$ (D) 0

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18. यदि $3P(A) = P(B) = \frac{3}{5}$ तथा $P(A \mid B) = \frac{2}{3}$ है, तो $P(A \cup B)$ है :

(A) $\frac{3}{5}$ (B) $\frac{1}{5}$

(C) $\frac{2}{15}$ (D) $\frac{2}{5}$

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं। दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है। इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए।*

(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है।

(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है।

19. *अभिकथन (A) :* समुच्चय {1, 2, 3} पर परिभाषित संबंध
$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$ एक तुल्यता संबंध है।

तर्क (R) : एक संबंध जो स्वतुल्य, सममित तथा संक्रामक है, वह एक तुल्यता संबंध होता है।

20. *अभिकथन (A) :* LPP : $Z = x + 2y$ का न्यूनतमीकरण करना, व्यवरोधों $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$ के अंतर्गत, पर विचार कीजिए। न्यूनतम Z अपरिमित रूप से अनेक बिंदुओं पर आता है। सुसंगत क्षेत्र के कोनीय बिंदु (0, 3) तथा (6, 0) हैं।

तर्क (R) : यदि दो कोनीय बिंदुओं पर उद्देश्य फलन का समान न्यूनतम मान आता है, तो इन बिंदुओं को मिलाने वाले रेखा-खंड के प्रत्येक बिंदु पर समान न्यूनतम मान आता है।

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18. If $3P(A) = P(B) = \frac{3}{5}$ and $P(A|B) = \frac{2}{3}$, then $P(A \cup B)$ is :

(A) $\frac{3}{5}$ (B) $\frac{1}{5}$

(C) $\frac{2}{15}$ (D) $\frac{2}{5}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* A relation R on the set {1, 2, 3} defined as R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)} is an equivalence relation.

Reason (R) : A relation that is reflexive, symmetric and transitive is an equivalence relation.

20. *Assertion (A) :* Consider a Linear Programming Problem with minimise $Z = x + 2y$ subject to constraints $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$ which gives minimum Z at infinitely many points. The corner points of feasible region are (0, 3) and (6, 0).

Reason (R) : If two corner points produce the same minimum value of the objective function, then every point on the line segment joining the points will give the same minimum value.

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खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के 5 प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं।

21. $\sin\left[\tan^{-1}\tan\left(\frac{3\pi}{4}\right)\right]$ का मान ज्ञात कीजिए।

22. (क) $x \log x$ के सापेक्ष x^x का अवकलन कीजिए।

अथवा

(ख) यदि $y = P\cos ux + Q\sin ux$ है, तो दर्शाइए कि $\frac{d^2y}{dx^2} + u^2y = 0$.

23. x के वह मान ज्ञात कीजिए जिनके लिए $f(x) = \frac{x-3}{x+1}$, $x \neq -1$, एक वर्धमान फलन है।

24. (क) तीन मधुमक्खियाँ क्रमश: सदिशों $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 4\hat{j} - 2\hat{k}$ तथा $\vec{c} = 3\hat{i} + 2\hat{k}$ की दिशा में उड़ती पाई गईं । λ का वह मान ज्ञात कीजिए ताकि $\vec{a} + \lambda\vec{b}$ का पथ $\vec{c}$ के लंबवत हो।

अथवा

(ख) यदि A, B तथा C तीन असरेखीय बिंदु इस प्रकार हैं कि $\overrightarrow{AB} = \hat{i} + 2\hat{j} - \hat{k}$ तथा $\overrightarrow{AC} = 2\hat{i} - 3\hat{j}$ है, तो त्रिभुज ABC का क्षेत्रफल ज्ञात कीजिए।

25. निम्नलिखित रेखाओं के युग्म के बीच का कोण ज्ञात कीजिए :

$$\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6} \text{ तथा}$$

$$\frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}$$

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Evaluate $\sin\left[\tan^{-1}\tan\left(\frac{3\pi}{4}\right)\right]$.

22. (a) Differentiate x^x with respect to $x \log x$.

OR

(b) If $y = P\cos ux + Q\sin ux$, show that $\frac{d^2y}{dx^2} + u^2y = 0$.

23. Determine the values of x for which $f(x) = \frac{x-3}{x+1}$, $x \neq -1$ is an increasing function.

24. (a) Three honey bees were found flying along the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 4\hat{j} - 2\hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{k}$ respectively. Find the value of λ such that the path for $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$.

OR

(b) If A, B and C be three non-collinear points such that $\overrightarrow{AB} = \hat{i} + 2\hat{j} - \hat{k}$ and $\overrightarrow{AC} = 2\hat{i} - 3\hat{j}$, then find the area of Δ ABC.

25. Find the angle between the following pair of lines :

$$\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6} \text{ and}$$

$$\frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}$$

❋

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के 6 प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. हवा निकलने के कारण एक गोलाकार गुब्बारे का आयतन इस प्रकार कम हो रहा है कि आयतन में कमी, किसी समय पर उसके पृष्ठीय क्षेत्रफल के समानुपाती है। दर्शाइए कि इसकी त्रिज्या एक निश्चित दर से कम हो रही है।

27. (क) ज्ञात कीजिए :

$$\int \frac{x - \sin x}{1 - \cos x}\, dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_0^2 \frac{1}{\sqrt{x^2 + 2x + 3}}\, dx$$

28. अवकल समीकरण $(x + 2y^3)\, dy = y\, dx$ का हल ज्ञात कीजिए।

29. ग्राफ द्वारा निम्नलिखित रैखिक प्रोग्रामन समस्या का हल ज्ञात कीजिए :
व्यवरोधों

$$2x + y \leq 1000$$
$$x + y \leq 800$$
$$x, y \geq 0$$

के अंतर्गत $Z = \frac{2x}{5} + \frac{3y}{10}$ का अधिकतमीकरण कीजिए।

30. (क) माना तीन खिलौने A, B तथा C एक सीधी रेखा में रखे गए हैं। यदि A, B तथा C के स्थिति सदिश क्रमश: $55\hat{i} - 2\hat{j}$, $5\hat{i} + 8\hat{j}$ तथा $a\hat{i} - 52\hat{j}$ हैं, तो 'a' का मान ज्ञात कीजिए।

अथवा

(ख) यदि $\vec{a}$, $\vec{b}$ तथा $\vec{c}$ मात्रक सदिश हैं, तो सिद्ध कीजिए कि

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 9.$$

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. A spherical balloon loses its volume due to escape of air from it in such a way that decrease of volume at any instant is proportional to its surface area. Show that the radius is decreasing at a constant rate.

27. (a) Find :

$$\int \frac{x - \sin x}{1 - \cos x}\, dx$$

OR

(b) Evaluate :

$$\int_0^2 \frac{1}{\sqrt{x^2 + 2x + 3}}\, dx$$

28. Solve the differential equation $(x + 2y^3)\, dy = y\, dx$.

29. Solve the following Linear Programming Problem graphically :

Maximize $Z = \frac{2x}{5} + \frac{3y}{10}$

subject to constraints

$$2x + y \leq 1000$$
$$x + y \leq 800$$
$$x, y \geq 0.$$

30. (a) Let three toys A, B and C be placed in the same straight line. If the position vectors of A, B and C are $55\hat{i} - 2\hat{j}$, $5\hat{i} + 8\hat{j}$ and $a\hat{i} - 52\hat{j}$ respectively, find the value of 'a'.

OR

(b) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then prove that

$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 9$.

❋

31. (क) एक पासे को उछाला गया। निम्न घटनाओं पर विचार कीजिए :

$A = \{1, 2, 5\}$, $B = \{3, 5\}$, $C = \{2, 3, 4, 5\}$

अत: निम्न ज्ञात कीजिए :

(i) $P(A \mid C)$ तथा $P(C \mid A)$

(ii) $P(A \cap B \mid C)$ तथा $P(A \cup B \mid C)$

अथवा

(ख) एक डिब्बे में 1 से 6 तक संख्यांकित 6 कार्ड हैं। एक विद्यार्थी को डिब्बे में से एक-एक करके प्रतिस्थापन सहित दो कार्ड निकालने तथा उन पर अंकित संख्याओं को नोट करने के लिए कहा गया। माना दोनों कार्डों पर आई संख्याओं का योगफल 10 आना घटना A है तथा पहले कार्ड पर 4 को छोड़ कर अन्य संख्या आना घटना B है।

P(A और B) ज्ञात कीजिए। अत: ज्ञात कीजिए कि क्या घटनाएँ A तथा B स्वतंत्र घटनाएँ हैं या नहीं।

खण्ड घ

इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. एक व्यक्ति बाज़ार से फल खरीदने जाता है, तो दुकानदार उसे बताता है कि 4 सेब, 3 संतरे तथा 2 केलों का मूल्य ₹ 60 है; 2 सेब, 4 संतरे तथा 6 केलों का मूल्य ₹ 90 है; जबकि 6 सेब, 2 संतरे तथा 3 केलों का मूल्य ₹ 70 है। आव्यूह विधि के प्रयोग से प्रत्येक प्रकार के एक-एक फल का मूल्य ज्ञात कीजिए।

33. (क) यदि $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$ है, तो दर्शाइए कि

$$(x^2 + 1)\frac{dy}{dx} + xy + 1 = 0.$$

अथवा

(ख) $x^{\cot x} + \dfrac{2x^2 - 3}{2x^2 - x + 2}$ का x के सापेक्ष अवकलज ज्ञात कीजिए।

31. (a) A die is rolled. Consider events :

$A = \{1, 2, 5\}$, $B = \{3, 5\}$, $C = \{2, 3, 4, 5\}$

and hence find :

(i) $P(A \mid C)$ and $P(C \mid A)$

(ii) $P(A \cap B \mid C)$ and $P(A \cup B \mid C)$

OR

(b) A box contains 6 cards numbered 1 to 6. A student is asked to pick up two cards, one by one after replacement and note down the numbers on the cards. Let A be the event of getting sum of the numbers on two cards as 10, and B, the event of a number other than 4 on the first card selected.

Find P(A and B) and find whether the events A and B are independent events or not.

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. A man goes to buy fruits from the market. The shopkeeper informs him that 4 apples, 3 oranges and 2 bananas cost ₹ 60; 2 apples, 4 oranges and 6 bananas cost ₹ 90; whereas 6 apples, 2 oranges and 3 bananas cost ₹ 70. Using matrix method, find the cost of one fruit of each kind.

33. (a) If $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$, show that

$$(x^2 + 1)\frac{dy}{dx} + xy + 1 = 0.$$

OR

(b) Find the differential of $x^{\cot x} + \frac{2x^2 - 3}{2x^2 - x + 2}$ with respect to x.

❋

34. वक्र $\{(x, y) : 100x^2 + 25y^2 = 2500\}$ का स्केच बनाइए तथा वक्र द्वारा परिबद्ध क्षेत्र का क्षेत्रफल, समाकलन विधि से ज्ञात कीजिए।

35. (क) बिंदु (0, 2, 3) से रेखा $\frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$ पर खींचे गए लंब का पाद ज्ञात कीजिए। अत: लंब की लंबाई ज्ञात कीजिए।

अथवा

(ख) यदि दो रेखाओं $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ तथा

$\vec{r} = (p\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

के बीच की न्यूनतम दूरी $\frac{3}{\sqrt{2}}$ इकाई है, तो p का मान ज्ञात कीजिए।

खण्ड ङ

इस खण्ड में 3 प्रकरण-अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36. एक स्कूल चाहता है कि कक्षा XII के विद्यार्थी दुनिया के पर्यावरण को ध्यान में रखते हुए 'सस्टेनेबिलिटी' पर एक प्रोजेक्ट करें। उन्होंने इसमें भाग लेने वाले विद्यार्थियों को एक निबंध लेखन प्रतियोगिता के आधार पर चुना।

इस प्रोजेक्ट के लिए 80 में से 7 विद्यार्थियों को चुना गया तथा उन्हें दो समुच्चयों में बाँटा गया :

समुच्चय A = $\{G_1, G_2, G_3, G_4\}$ लड़कियों का समुच्चय है, तथा

समुच्चय B = $\{B_1, B_2, B_3\}$ लड़कों का समुच्चय है।

उपर्युक्त सूचना के आधार पर निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) समुच्चय A $\rightarrow$ समुच्चय B तक कितने संबंध संभव हैं ? *1*

34. Sketch the curve $\{(x, y) : 100x^2 + 25y^2 = 2500\}$ and find the area of the region enclosed by it, using integration.

35. (a) Find the foot of the perpendicular from the point (0, 2, 3) on the line $\frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$ and hence find the length of the perpendicular.

OR

(b) Find the value of p if the shortest distance between the lines

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and

$\vec{r} = (p\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

is $\frac{3}{\sqrt{2}}$ units.

SECTION E

This section comprises 3 Case Study based questions of 4 marks each.

Case Study – 1

36. A school wants the students of class XII to do a project on 'Sustainability' keeping the world environment in mind. They select the student participants on the basis of an essay writing competition.

7 students out of 80 are selected for the project and are categorized into two sets such that :

Girl students belong to Set A = $\{G_1, G_2, G_3, G_4\}$,

Boy students belong to Set B = $\{B_1, B_2, B_3\}$.

Based on the above information, answer the following questions :

(i) How many relations are possible from Set A $\rightarrow$ Set B ? *1*

(ii) माना संबंध $R : A \to B$ इस प्रकार है कि

$R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$ है।

क्या R एक एकैकी फलन है ? अपने उत्तर की पुष्टि कीजिए। *1*

(iii) (क) माना संबंध $R : A \to A$ इस प्रकार परिभाषित है कि

$R = \{(x, y), x, y \in A,$ x तथा y शहर की एक ही कॉलोनी के विद्यार्थी हैं $\}$

जाँच कीजिए कि क्या R एक तुल्यता संबंध है। *2*

अथवा

(iii) (ख) जाँच कीजिए कि क्या कोई फलन $f : B \to A$ एकैकी आच्छादी है। अपने उत्तर के समर्थन में कारण दीजिए। *2*

प्रकरण अध्ययन – 2

37. देश के लोगों को एक विशेष प्रकार के इंफेक्शन से बचाने के लिए मार्केट में तीन तरह की वैक्सीन A_1, A_2 तथा A_3 उपलब्ध हैं। एक सर्वेक्षण के अनुसार यह पाया गया कि 25% आबादी को वैक्सीन A_1, 35% आबादी को वैक्सीन A_2 तथा 40% आबादी को वैक्सीन A_3 दी गई तथा यह भी पाया गया कि वैक्सीन A_1, A_2 तथा A_3 की इंफेक्शन से बचाव करने की प्रायिकताएँ क्रमश: 60%, 55% तथा 50% थीं।

उपर्युक्त सूचना के आधार पर निम्नलिखित प्रश्नों के उत्तर दीजिए :

प्रायिकता ज्ञात कीजिए कि

(i) एक A_2 वैक्सीन लेने वाले व्यक्ति को इंफेक्शन हो जाए। *1*

(ii) एक यादृच्छया चुने गए व्यक्ति का इंफेक्शन से बचाव हो जाए। *1*

(ii) Let R be a relation from A $\rightarrow$ B such that

$R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$.

Is R an injective function ? Justify your answer. *1*

(iii) (a) Let the relation R from A $\rightarrow$ A be such that

R = {(x, y), x, y $\in$ A, x and y are students from the same colony in the city}

Verify if R is an equivalence relation. *2*

OR

(iii) (b) Verify if any function f : B $\rightarrow$ A is bijective. Give reason to support your answer. *2*

Case Study – 2

37. There are three types of vaccines A_1, A_2, A_3, available in the market to protect the population of the country from spread of certain infection. According to a survey conducted, it was found that 25% of the population was given Vaccine A_1, 35% of the population was given Vaccine A_2 and 40% of the population was given Vaccine A_3. The survey also stated that the probabilities that Vaccines A_1, A_2 and A_3 would protect against the infection were 60%, 55% and 50% respectively.

Based on the above information, answer the following questions :

Find the probability that :

(i) The person taking vaccine A_2 will get infected. *1*

(ii) If a person is chosen randomly, he/she will be protected from the infection. *1*

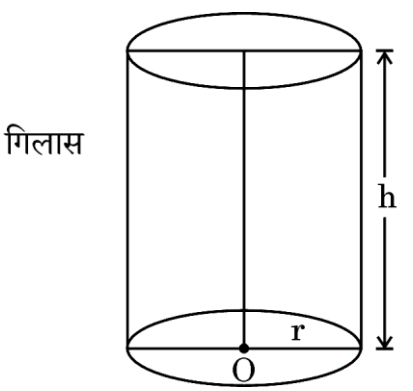
(iii) (क) एक यादृच्छया चुना गया व्यक्ति इंफेक्शन से ग्रसित पाया गया, तो उसने वैक्सीन A_1 ली थी। *2*

अथवा

(iii) (ख) एक यादृच्छया चुना गया व्यक्ति इंफेक्शन से मुक्त पाया गया, तो उसे वैक्सीन A_3 दी गई थी। *2*

प्रकरण अध्ययन – 3

38. एक कंपनी ऊपर से खुले बेलनाकार गिलास बनाती है। उत्पाद में एकरूपता बनाए रखने के लिए उन्होंने बनाए गए गिलासों का पृष्ठीय क्षेत्रफल निश्चित कर दिया।



उपर्युक्त सूचना के आधार पर निम्नलिखित प्रश्नों के उत्तर दीजिए :

यदि एक गिलास का आयतन V, ऊँचाई h तथा वृत्तीय आधार की त्रिज्या r है, तो :

(i) इसके आयतन का आधार की त्रिज्या के सापेक्ष अवकलज ज्ञात कीजिए, जहाँ पृष्ठीय क्षेत्रफल नियत है। *2*

(ii) यदि यह कंपनी प्रत्येक गिलास के आयतन का अधिकतमीकरण करना चाहती है, तो ऊँचाई और आधार की त्रिज्या में संबंध स्थापित कीजिए। *2*

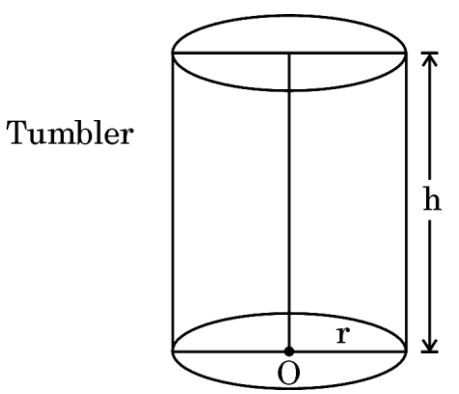
(iii) (a) The person was given Vaccine A_1, given that the randomly chosen person is infected. *2*

OR

(iii) (b) The person was given Vaccine A_3, given that the randomly chosen person is not infected. *2*

Case Study – 3

38. A company produces cylindrical tumblers, open from the top. Since they want uniformity in the product, they fix the surface area of the tumblers produced.



Based on the above information, answer the following questions :

If for a tumbler, V is its volume, h the height and r the radius of the circular base, then :

(i) Differentiate its volume with respect to radius of the base, where the surface area is constant. *2*

(ii) If the company wants to maximize the volume of each tumbler, then establish a relation between its height and the radius of the base. *2*

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary Certificate Examination, 2026
MATHEMATICS (041) (PAPER CODE 65/5/1)

General Instructions: -

1.	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2.	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and BNS."**
3.	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, Answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given Answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4.	The Marking Scheme carries only suggested value points for the Answers. These are Guidelines only and do not constitute the complete Answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5.	The Head-Examiner must go through the first five Answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining Answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6.	Evaluators will mark (√) wherever Answer is correct. For wrong Answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the Answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7.	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left- hand margin and encircled. This may be followed strictly.
8.	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9.	If a student has attempted an extra question, Answer to the question deserving more marks should be retained and the other Answer scored out with a note **"Extra Question"**.
10.	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11.	A full scale of marks__ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the Answer deserves it.

12.	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 Answer books per day in main subjects and 25 Answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13.	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving Answer or part thereof unassessed in an Answer book. ● Giving more marks for an Answer than assigned to it. ● Wrong totaling of marks awarded on an Answer. ● Wrong transfer of marks from the inside pages of the Answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the Answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect Answer.) ● Half or a part of the Answer marked correct and the rest as wrong, but no marks awarded.
14.	While evaluating the Answer books if the Answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15.	Any unassessed portion, non-carrying over marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16.	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17.	Every Examiner shall also ensure that all the Answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18.	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each Answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/5/1)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Steps	Marks
	SECTION A Q. Number 1 to 20 are multiple choice questions of 1 mark each.		
1.	If matrix $A = \begin{bmatrix} -p & q \\ r & p \end{bmatrix}$ is such that $A^2 = I$, then : (A) $1 + p^2 + qr = 0$ (B) $1 - p^2 - qr = 0$ (C) $1 - p^2 + qr = 0$ (D) $1 + p^2 - qr = 0$		
Sol.	(B) $1 - p^2 - qr = 0$		1
2.	If A is a square matrix such that $A^2 = A$, then $(A - I)^3 - A$ is equal to : (A) I (B) $-I$ (C) A (D) A^2		
Sol.	(B) $-I$		1
3.	For the inverse trigonometric functions, which of the following Principal Value Branch is ***not*** correctly defined ? (A) $\tan^{-1} : R \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (B) $\sec^{-1} : R - (-1, 1) \to [0, \pi] - \left\{\frac{\pi}{2}\right\}$ (C) $\cot^{-1} : R \to (0, \pi)$ (D) $\text{cosec}^{-1} : R - (-1, 1) \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$		
Sol.	(D) $\text{cosec}^{-1} : R - (-1, 1) \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$		1
4.	Let $A = \begin{bmatrix} 0 & -3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix}$. If $A + B + C = O$, then matrix C is : (A) $\begin{bmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 3 & 3 & 5 \\ -3 & -4 & -2 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} -3 & -3 & -5 \\ 3 & 4 & 2 \end{bmatrix}$		

Sol.	(C) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$		1
5.	If A is a non-singular matrix, then which of the following is ***not*** true ? (A) adj A is singular (B) $(\text{adj } A)^{-1} = (\text{adj } A^{-1})$ (C) $\lvert A \rvert \neq 0$ (D) A^{-1} exists		
Sol.	(A) adj A is singular		1
6.	If $f(x) = \begin{cases} \dfrac{x^2-4x-5}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$ is continuous at $x = -1$, then the value of k is : (A) Any real value (B) 6 (C) -1 (D) -6		
Sol.	(D) -6		1
7.	If the area of Δ ABC with vertices A(3, 1), B(– 2, 1) and C(0, k) is 5 sq. units, then values of k are : (A) 3, 1 (B) $-1, 3$ (C) $-1, 2$ (D) 0, 2		
Sol.	(B) $-1, 3$		1
8.	Derivative of $\cos^{-1}\left(\dfrac{\sin x + \cos x}{\sqrt{2}}\right)$, $-\dfrac{\pi}{4} < x < \dfrac{\pi}{4}$ with respect to x is : (A) -1 (B) 1 (C) $\dfrac{\pi}{4}$ (D) $-\dfrac{\pi}{4}$		
Sol.	(A) -1		1
9.	Absolute minimum value of $f(x) = (x-2)^2 + 5$ in the interval $[-3, 2]$ is : (A) -3 (B) 2 (C) 5 (D) 30		
Sol.	(C) 5		1

10.	$\int \frac{1}{\sqrt{1+\cos 2x}} dx$ is equal to : (A) $\log \cos x + C$ (B) $\frac{1}{\sqrt{2}} \log \lvert \sec x + \tan x \rvert + C$ (C) $\frac{1}{\sqrt{2}} \log \lvert \sec x - \tan x \rvert + C$ (D) $\log \sin 2x + C$		
Sol.	(B) $\frac{1}{\sqrt{2}} \log \lvert \sec x + \tan x \rvert + C$		1
11.	The value of $\int_{-5}^{-1} \frac{1}{x} dx$ is equal to : (A) $-\log 5$ (B) x^6 (C) $\log(-5)$ (D) x^{-6}		
Sol.	(A) $-\log 5$		1
12.	An ant is observed crawling on a sheet of paper along a straight line given by equation $y = 2x - 4$. Area of the surface covered by the ant bounded by y-axis, x-axis and $x = 1$ is : (A) 1 sq. unit (B) 3 sq. units (C) 2 sq. units (D) 4 sq. units		
Sol.	(B) 3 sq. units		1
13.	The order and degree of the differential equation $1 + \left(\frac{d^3y}{dx^3}\right)^3 = \lambda \frac{d^2y}{dx^2}$ is : (A) Order = 3, Degree = 3 (B) Order = 2, Degree = 2 (C) Order = 3, Degree = 1 (D) Order = 2, Degree = 1		
Sol.	(A) Order = 3, Degree = 3		1
14.	The general solution for the differential equation $\frac{dy}{dx} = e^{3x-y}$ is : (A) $3e^y = e^{3x} + C$ (B) $\log(3x - y) = C$ (C) $e^{3x-y} = C$ (D) $-e^y + 3e^{3x} = C$		
Sol.	(A) $3e^y = e^{3x} + C$		1

15.	The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40) (60, 20) and (60, 0). If the objective function of an LPP is Z = 4x + 3y, then the maximum value is : (A) 200 (B) 300 (C) 240 (D) 120		
Sol.	(B) 300		1
16.	If position vector $\vec{p}$ of a point (24, n) is such that $\lvert \vec{p} \rvert = 25$, then the value of n is : (A) ± 49 (B) ± 5 (C) ± 1 (D) ± 7		
Sol.	(D) ± 7		1
17.	If vectors $\vec{a} = 3\hat{i} + 2\hat{j} + \lambda\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$, represent the two strips of the Red Cross sign placed outside a doctor's clinic, then the value of λ is : (A) 1 (B) $\frac{5}{2}$ (C) $\frac{2}{5}$ (D) 0		
Sol.	(C) $\frac{2}{5}$		1
18.	If $3P(A) = P(B) = \frac{3}{5}$ and $P(A \mid B) = \frac{2}{3}$, then $P(A \cup B)$ is : (A) $\frac{3}{5}$ (B) $\frac{1}{5}$ (C) $\frac{2}{15}$ (D) $\frac{2}{5}$		
Sol.	(D) $\frac{2}{5}$		1

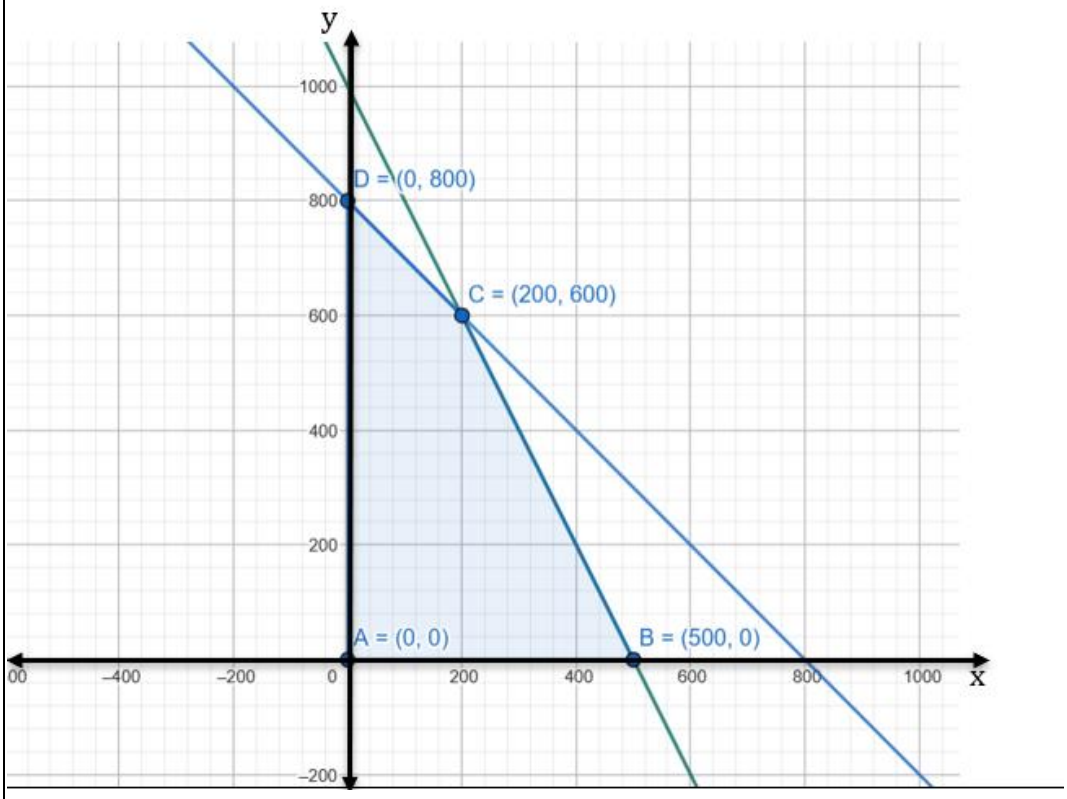
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.		
19.	*Assertion (A) :* A relation R on the set {1, 2, 3} defined as R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)} is an equivalence relation. *Reason (R) :* A relation that is reflexive, symmetric and transitive is an equivalence relation.		
Sol.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
20.	*Assertion (A) :* Consider a Linear Programming Problem with minimise Z = x + 2y subject to constraints 2x + y ≥ 3, x + 2y ≥ 6, x, y ≥ 0 which gives minimum Z at infinitely many points. The corner points of feasible region are (0, 3) and (6, 0). *Reason (R) :* If two corner points produce the same minimum value of the objective function, then every point on the line segment joining the points will give the same minimum value.		
Sol.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).		1
	SECTION B Q. Numbers 21 to 25 are very short answer questions of 2 marks each.		
21.	Evaluate $\sin\left[\tan^{-1}\tan\left(\frac{3\pi}{4}\right)\right]$.		
Sol.	Given expression = $\sin[\tan^{-1}(-1)]$ or $\sin[\tan^{-1}\tan(\pi - \frac{\pi}{4})]$ $= \sin\left(-\frac{\pi}{4}\right)$ $= -\frac{1}{\sqrt{2}}$	I II III	1 ½ ½

22.	(a) Differentiate x^x with respect to $x \log x$. **OR** (b) If $y = P \cos ux + Q \sin ux$, show that $\frac{d^2y}{dx^2} + u^2y = 0$.		
Sol.	(a) Let $u = x^x$ and $v = x \log x$	I	½
	$\frac{du}{dx} = x^x(1 + \log x)$	II	½
	$\frac{dv}{dx} = 1 + \log x$	III	½
	$\therefore \frac{du}{dv} = x^x$	IV	½
	OR		
	(b) $\frac{dy}{dx} = -P u \sin ux + Q u \cos ux$	I	½
	$\frac{d^2y}{dx^2} = -P u^2 \cos ux - Q u^2 \sin ux$	II	½
	$\frac{d^2y}{dx^2} = -u^2(P \cos ux + Q \sin ux)$	III	½
	$\therefore \frac{d^2y}{dx^2} + u^2y = 0$	IV	½
23.	Determine the values of x for which $f(x) = \frac{x-3}{x+1}$, $x \neq -1$ is an increasing function.		
Sol.	$f'(x) = \frac{(x+1).1-(x-3).1}{(x+1)^2}$	I	1
	$= \frac{4}{(x+1)^2} > 0$	II	½
	$\therefore$ f(x) is an increasing function for all $x \in R - \{-1\}$	III	½

24.	(a) Three honey bees were found flying along the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 4\hat{j} - 2\hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{k}$ respectively. Find the value of λ such that the path for $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$. **OR** (b) If A, B and C be three non-collinear points such that $\overrightarrow{AB} = \hat{i} + 2\hat{j} - \hat{k}$ and $\overrightarrow{AC} = 2\hat{i} - 3\hat{j}$, then find the area of Δ ABC.		

Sol.	As $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$ $\therefore (\vec{a} + \lambda\vec{b}) . \vec{c} = 0$	I	½
	i.e. $\vec{a}.\vec{c} + \lambda\vec{b}.\vec{c} = 0$		
	which gives $8 + \lambda(-4) = 0$	II	1
	$\lambda = 2$	III	½
	OR		
	Area of $\Delta ABC = \frac{1}{2}\lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert$		
	$= \frac{1}{2}\lvert -3\hat{i} - 2\hat{j} - 7\hat{k} \rvert$	I	1
	$= \frac{\sqrt{62}}{2}$	II	1
25.	Find the angle between the following pair of lines : $\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6}$ and $\frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}$		
Sol.	Drs of given lines are <3, 2, 6> and <1, 2, 2 > Let θ be angle between given lines	I	½
	$\therefore \cos\theta = \frac{3\times1+2\times2+6\times2}{7\times3}$	II	1
	Thus, $\theta = \cos^{-1}\left(\frac{19}{21}\right)$	III	½
	SECTION C Q. Numbers 26 to 31 are short answer questions of 3 marks each.		

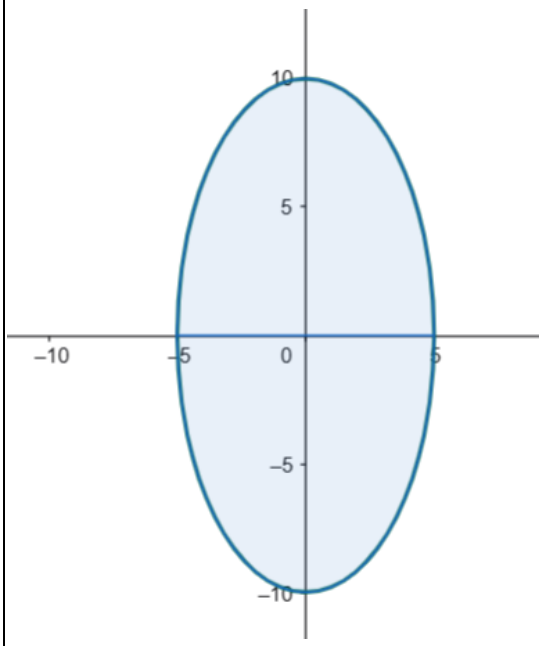
26.	A spherical balloon loses its volume due to escape of air from it in such a way that decrease of volume at any instant is proportional to its surface area. Show that the radius is decreasing at a constant rate.		
Sol.	Let r be the radius, V= $\frac{4}{3}\pi r^3$ be the volume $\frac{dV}{dt} = -k(4\pi r^2)$ where $k > 0$ $4\pi r^2 \frac{dr}{dt} = -4k\pi r^2$ $\therefore \frac{dr}{dt} = -k$ which is a constant Thus, radius is decreasing at a constant rate	I II III	1 1 1
27.	(a) Find : $\int \frac{x - \sin x}{1 - \cos x}\, dx$ **OR** (b) Evaluate : $\int_0^2 \frac{1}{\sqrt{x^2 + 2x + 3}}\, dx$		
Sol.	(a) Let $I = \int \frac{x - \sin x}{1 - \cos x} dx$ $= \int \frac{x}{1 - \cos x} dx - \int \frac{\sin x}{1 - \cos x} dx$ $= \int \frac{x}{2\sin^2 \frac{x}{2}} dx - \int \frac{2\sin \frac{x}{2} \cos \frac{x}{2}}{2\sin^2 \frac{x}{2}}\, dx$ $= -\frac{x}{2}.2\cot \frac{x}{2} - \frac{1}{2}\int -2\cot \frac{x}{2} dx - \int \cot \frac{x}{2}\, dx$ $= -x \cot \frac{x}{2} + C$ **OR** (b) $I = \int_0^2 \frac{1}{\sqrt{x^2 + 2x + 3}} dx$ $= \int_0^2 \frac{1}{\sqrt{(x+1)^2 + 2}} dx$ $= \left[\log (x + 1 + \sqrt{x^2 + 2x + 3})\right]_0^2$ $= \log (3 + \sqrt{11}) - \log (1 + \sqrt{3})$ or $\log \left(\frac{3 + \sqrt{11}}{1 + \sqrt{3}}\right)$	I II III IV I II III	½ 1 1 ½ 1 1 1
28.	Solve the differential equation $(x + 2y^3)\, dy = y\, dx$.		
Sol.	Given differential equation can be written as		

	$\frac{dx}{dy} - \frac{x}{y} = 2y^2$	I	½
	Integrating factor = $e^{\int -\frac{1}{y} dy} = e^{-\log y} = \frac{1}{y}$	II	1
	Solution is $x \cdot \frac{1}{y} = \int 2y \, dy + C$	III	1
	i.e., $x \cdot \frac{1}{y} = y^2 + C$ or $x = y^3 + Cy$	IV	½
29.	**Solve the following Linear Programming Problem graphically :** **Maximize** $Z = \frac{2x}{5} + \frac{3y}{10}$ **subject to constraints** $2x + y \leq 1000$ $x + y \leq 800$ $x, y \geq 0.$		
Sol		I	Correct Lines and Shading 1½
	Corner Points — Value of Z (0, 0) — 0 (500, 0) — 200 (200, 600) — 260 (0, 800) — 240	II	1
	Z_{max} = 260 at x = 200, y = 600	III	½

30.	(a) Let three toys A, B and C be placed in the same straight line. If the position vectors of A, B and C are $55\hat{i} - 2\hat{j}$, $5\hat{i} + 8\hat{j}$ and $a\hat{i} - 52\hat{j}$ respectively, find the value of 'a'. **OR** (b) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors, then prove that $\lvert \vec{a} - \vec{b} \rvert^2 + \lvert \vec{b} - \vec{c} \rvert^2 + \lvert \vec{c} - \vec{a} \rvert^2 \leq 9$.		
Sol.	(a) $\overrightarrow{AB} = -50\hat{i} + 10\hat{j}$ and $\overrightarrow{BC} = (a-5)\hat{i} - 60\hat{j}$ As $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are collinear vectors $\therefore \frac{-50}{a-5} = \frac{10}{-60}$ $a = 305$ **OR** (b) $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert = 1$ Now, consider $\lvert\vec{a} - \vec{b}\rvert^2 + \lvert\vec{b} - \vec{c}\rvert^2 + \lvert\vec{c} - \vec{a}\rvert^2$ $= 2\lvert\vec{a}\rvert^2 + 2\lvert\vec{b}\rvert^2 + 2\lvert\vec{c}\rvert^2 - 2\vec{a}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{c}.\vec{a}$ $= 3\lvert\vec{a}\rvert^2 + 3\lvert\vec{b}\rvert^2 + 3\lvert\vec{c}\rvert^2 - (\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 + \lvert\vec{c}\rvert^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a})$ $= 9 - \lvert\vec{a} + \vec{b} + \vec{c}\rvert^2 \leq 9$ $\lvert\vec{a} - \vec{b}\rvert^2 + \lvert\vec{b} - \vec{c}\rvert^2 + \lvert\vec{c} - \vec{a}\rvert^2 \leq 9$	I II III I II III IV	1 1 1 ½ 1½ ½ ½
31.	(a) A die is rolled. Consider events : A = {1, 2, 5}, B = {3, 5}, C = {2, 3, 4, 5} and hence find : (i) P(A | C) and P(C | A) (ii) P(A ∩ B | C) and P(A ∪ B | C) **OR** (b) A box contains 6 cards numbered 1 to 6. A student is asked to pick up two cards, one by one after replacement and note down the numbers on the cards. Let A be the event of getting sum of the numbers on two cards as 10, and B, the event of a number other than 4 on the first card selected. Find P(A and B) and find whether the events A and B are independent events or not.		
Sol.	(a) (i) $P(A\|C) = \frac{n(A \cap C)}{n(C)} = \frac{2}{4}$ or $\frac{1}{2}$	I	1

	and $P(C\|A) = \frac{n(C \cap A)}{n(A)} = \frac{2}{3}$	II	½
	(ii) $P(A \cap B\|C) = \frac{n(A \cap B \cap C)}{n(C)} = \frac{1}{4}$	III	1
	and $P(A \cup B\|C) = \frac{n[(A \cup B) \cap C]}{n(C)} = \frac{3}{4}$	IV	½
	OR		
	(b) $A = \{(4, 6), (6, 4), (5, 5)\}$ or $n(A) = 3$	I	½
	$n(B) = 30$		
	$n(A \cap B) = 2$	II	½
	$P(A \cap B) = \frac{2}{36}$	III	½
	$P(A) = \frac{3}{36} = \frac{1}{12}$		
	$P(B) = \frac{30}{36} = \frac{5}{6}$	IV	½
	$P(A).P(B) = \frac{5}{72} \neq P(A \cap B)$	V	½
	Hence A and B are not independent	VI	½
	SECTION D Q. Numbers 32 to 35 are long answer questions of 5 marks each.		
32.	A man goes to buy fruits from the market. The shopkeeper informs him that 4 apples, 3 oranges and 2 bananas cost ₹ 60; 2 apples, 4 oranges and 6 bananas cost ₹ 90; whereas 6 apples, 2 oranges and 3 bananas cost ₹ 70. Using matrix method, find the cost of one fruit of each kind.		
Sol.	Let cost of one apple, one orange, one banana be ₹ x, ₹ y and ₹ z respectively According to the question, $4x + 3y + 2z = 60$ $2x + 4y + 6z = 90$ $6x + 2y + 3z = 70$	I	1
	Let $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$		
	$\|A\| = 50 \neq 0 \Rightarrow A^{-1}$ exists	II	1
	System becomes AX = B. So, $X = A^{-1}B$		
	$adjA = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$	III	1½
	$A^{-1} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$	IV	½
	$X = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$		
	$X = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$	V	1

	∴ cost of one apple, one orange, one banana is ₹ 5, ₹ 8 and ₹ 8 respectively		
33.	(a) If $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$, show that $(x^2+1)\frac{dy}{dx} + xy + 1 = 0.$ **OR** (b) Find the differential of $x^{\cot x} + \frac{2x^2-3}{2x^2-x+2}$ with respect to x.		
Sol.	(a) Differentiating both sides with respect to x, we get $\sqrt{x^2+1}\frac{dy}{dx} + y.\frac{x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}.\frac{x}{\sqrt{x^2+1}} - 1$ $(x^2+1)\frac{dy}{dx} + y.x = \frac{x}{\sqrt{x^2+1}} - \sqrt{x^2+1}$ Note: In case student, considers $\sqrt{x^2+1} - x$ as part of log and writes following solution Differentiating both sides with respect to x, we get $\sqrt{x^2+1}\frac{dy}{dx} + y.\frac{x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}-x}.\left(\frac{x}{\sqrt{x^2+1}} - 1\right)$ $\sqrt{x^2+1}\frac{dy}{dx} + y.\frac{x}{\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}-x}.\left(\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right)$ $\sqrt{x^2+1}\frac{dy}{dx} + y.\frac{x}{\sqrt{x^2+1}} = \frac{-1}{\sqrt{x^2+1}}$ gives $(x^2+1)\frac{dy}{dx} + y.x = -1$ or $(x^2+1)\frac{dy}{dx} + x.y + 1 = 0$ **OR** (b) Let $u = x^{\cot x}$ and $v = \frac{2x^2-3}{2x^2-x+2}$ $\log u = \cot x.\log x$ $\frac{1}{u}\frac{du}{dx} = \frac{\cot x}{x} + \log x(-\operatorname{cosec}^2 x)$ $\frac{du}{dx} = x^{\cot x}\left(\frac{\cot x}{x} - \operatorname{cosec}^2 x.\log x\right)$ $\frac{dv}{dx} = \frac{(2x^2-x+2)(4x)-(2x^2-3)(4x-1)}{(2x^2-x+2)^2}$ $= \frac{-2x^2+20x-3}{(2x^2-x+2)^2}$ Required derivative $= x^{\cot x}\left(\frac{\cot x}{x} - \operatorname{cosec}^2 x.\log x\right) + \frac{-2x^2+20x-3}{(2x^2-x+2)^2}$	I II I II III IV I II III IV V VI VII	2 + 2 1 2 1 1 1 ½ ½ 1 ½ 1 1 ½

34.	Sketch the curve $\{(x, y) : 100x^2 + 25y^2 = 2500\}$ and find the area of the region enclosed by it, using integration.		
Sol.		I	1½
	Required area $= 4\int_0^5 \frac{10}{5}\sqrt{25-x^2}\,dx$	II	1½
	$= 8\left[\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_0^5$	III	1
	$= 50\pi$	IV	1
35.	(a) Find the foot of the perpendicular from the point (0, 2, 3) on the line $\frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$ and hence find the length of the perpendicular. **OR** (b) Find the value of p if the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (p\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ is $\frac{3}{\sqrt{2}}$ units.		

Sol.	(a) Given line is $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$	I	1
	Any general point on line l is $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$	II	½
	Drs of given line are <5, 2, 3>		
	Drs of perpendicular line $are < 5\lambda - 3, 2\lambda - 1, 3\lambda - 7 >$	III	½
	As lines are perpendicular		
	$\therefore 5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$ gives, $\lambda = 1$	IV	1
	$\therefore$ coordinates of foot of perpendicular are $(2, 3, -1)$	V	1
	length of perpendicular $= \sqrt{21}$	VI	1
	OR		
	(b) Let $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{a}_2 = p\hat{i} - \hat{j} - \hat{k}$, and		
	$\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$		
	Here $(\vec{a}_2 - \vec{a}_1) = (p-1)\hat{i} - 3\hat{j} - 2\hat{k}$	I	1
	$\vec{b}_1 \times \vec{b}_2 = -3\hat{i} + 3\hat{k}$	II	1½
	S.D. $= \left\| \frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{\lvert\vec{b}_1 \times \vec{b}_2\rvert} \right\|$		
	$\frac{-3p-3}{3\sqrt{2}} = \pm \frac{3}{\sqrt{2}}$	III	1½
	gives p = $-4, 2$	IV	1
	SECTION E This section (Q. 36 to 38) has 3 case study based questions of 4 marks each.		
36.	**Case Study – 1** A school wants the students of class XII to do a project on 'Sustainability' keeping the world environment in mind. They select the student participants on the basis of an essay writing competition. 7 students out of 80 are selected for the project and are categorized into two sets such that : Girl students belong to Set A = $\{G_1, G_2, G_3, G_4\}$, Boy students belong to Set B = $\{B_1, B_2, B_3\}$. Based on the above information, answer the following questions : (i) How many relations are possible from Set A → Set B ? *1*		

	(ii) Let R be a relation from A → B such that $R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$. Is R an injective function ? Justify your answer. *1* (iii) (a) Let the relation R from A → A be such that R = {(x, y), x, y ∈ A, x and y are students from the same colony in the city} Verify if R is an equivalence relation. *2* **OR** (iii) (b) Verify if any function f : B → A is bijective. Give reason to support your answer. *2*		
Sol.	(i) Number of relations from A to B is 2^{12} or 4096	I	1
	(ii) No R is not injective	I	½
	It is not a function as G_1 has two images B_1 and B_2	II	½
	(iii) (a) R is reflexive as (x, x) ∈ R ∀ ∈ *A* because x and x are students from same colony	I	½
	Let (x, y) ∈ R so x and y are students from same colony Hence y and x are students from same colony Thus, (y, x) ∈ R ∴ R is symmetric	II	½
	Let (x, y) ∈ R and (y, z) ∈ R so x and y are students from same colony, y and z are students from same colony Thus, x and z are students from same colony ∴ (x, z) ∈ R Thus, R is transitive	III	½
	As R is reflexive, symmetric and transitive, hence R is an equivalence relation	IV	½
	OR		
	(iii) (b) Any function f from B to A will not be bijective	I	1
	As f from B to A cannot be surjective because n(B) < n(A) or A and B are finite sets and n(A) ≠ n(B)	II	1

37. **Case Study – 2**

There are three types of vaccines A_1, A_2, A_3, available in the market to protect the population of the country from spread of certain infection. According to a survey conducted, it was found that 25% of the population was given Vaccine A_1, 35% of the population was given Vaccine A_2 and 40% of the population was given Vaccine A_3. The survey also stated that the probabilities that Vaccines A_1, A_2 and A_3 would protect against the infection were 60%, 55% and 50% respectively.

Based on the above information, answer the following questions :

Find the probability that :

(i) The person taking vaccine A_2 will get infected. *1*

(ii) If a person is chosen randomly, he/she will be protected from the infection. *1*

(iii) (a) The person was given Vaccine A_1, given that the randomly chosen person is infected. *2*

OR

(iii) (b) The person was given Vaccine A_3, given that the randomly chosen person is not infected. *2*

Sol.

(i) P (Person taking vaccine A_2 will be infected) = 45% or 0.45 — I — 1

(ii) P (Person is protected from infection)

$$= \frac{25}{100}.\frac{60}{100} + \frac{35}{100}.\frac{55}{100} + \frac{40}{100}.\frac{50}{100}$$ — I — ½

$$= \frac{5425}{10000} \text{ or } 0.5425$$ — II — ½

(iii) (a) P (A_1/person is infected) $= \dfrac{\frac{25}{100}.\frac{40}{100}}{\frac{25}{100}.\frac{40}{100}+\frac{35}{100}.\frac{45}{100}+\frac{40}{100}.\frac{50}{100}}$ — I — 1½

$$= \frac{1000}{4575} \text{ or } \frac{40}{183}$$ — II — ½

OR

(iii) (b) P(A_3/person is not infected) $= \dfrac{\frac{40}{100}.\frac{50}{100}}{\frac{25}{100}.\frac{60}{100}+\frac{35}{100}.\frac{55}{100}+\frac{40}{100}.\frac{50}{100}}$ — I — 1½

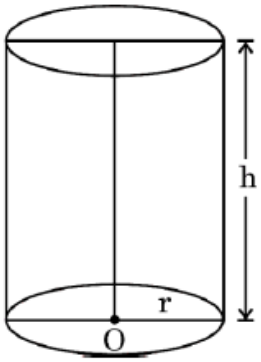
$$= \frac{2000}{5425} \text{ or } \frac{80}{217}$$ — II — ½

38.

Case Study – 3

A company produces cylindrical tumblers, open from the top. Since they want uniformity in the product, they fix the surface area of the tumblers produced.

Tumbler



Based on the above information, answer the following questions :

If for a tumbler, V is its volume, h the height and r the radius of the circular base, then :

(i) Differentiate its volume with respect to radius of the base, where the surface area is constant. *2*

(ii) If the company wants to maximize the volume of each tumbler, then establish a relation between its height and the radius of the base. *2*

Sol.

(i) Surface area $S = 2\pi rh + \pi r^2$ — I — ½

$V = \pi r^2 h = \frac{\pi r^2(S - \pi r^2)}{2\pi r} = \frac{(Sr - \pi r^3)}{2}$ — II — 1

$\frac{dV}{dr} = \frac{(S - 3\pi r^2)}{2}$ — III — ½

(ii) $\frac{dV}{dr} = 0$ gives $S = 3\pi r^2$ — I — 1

$\frac{d^2V}{dr^2} = -3\pi r < 0$ — II — ½

V is maximum when $S = 3\pi r^2$

Thus, $h = \frac{3\pi r^2 - \pi r^2}{2\pi r} = r$ — III — ½

Series : Z6YWX

SET~1

प्रश्न-पत्र कोड
Q.P. Code **65/6/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

निर्धारित समय : ***3*** *घण्टे*
Time allowed : ***3*** *hours*

अधिकतम अंक : ***80***
Maximum Marks : ***80***

नोट	**NOTE**
(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।	(I) Please check that this question paper contains **23** printed pages.
(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।	(II) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।	(III) Please check that this question paper contains **38** questions.
(IV) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।**	(IV) **Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**
(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।	(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में **38** प्रश्न हैं। **सभी** प्रश्न **अनिवार्य** हैं।*

(ii) *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ**।*

(iii) ***खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय (MCQ) तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **1** अंक के प्रश्न हैं।*

(iv) ***खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **2** अंकों के प्रश्न हैं।*

(v) ***खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **3** अंकों के प्रश्न हैं।*

(vi) ***खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **5** अंकों के प्रश्न हैं।*

(vii) ***खण्ड ङ** में प्रश्न संख्या **36** से **38** तक प्रकरण अध्ययन आधारित **4** अंकों के प्रश्न हैं।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है। यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न (MCQ) हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है।

1. माना आव्यूह A और B के लिए AB' और $B'A$ दोनों परिभाषित हैं। यदि आव्यूह A की कोटि $n \times m$ है, तो आव्यूह B की कोटि है :

(A) $n \times n$ (B) $n \times m$

(C) $m \times m$ (D) $m \times n$

2. यदि $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ है, तो A एक :

(A) अदिश आव्यूह है (B) तत्समक आव्यूह है

(C) सममित आव्यूह है (D) विषम-सममित आव्यूह है

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is :

(A) $n \times n$ (B) $n \times m$

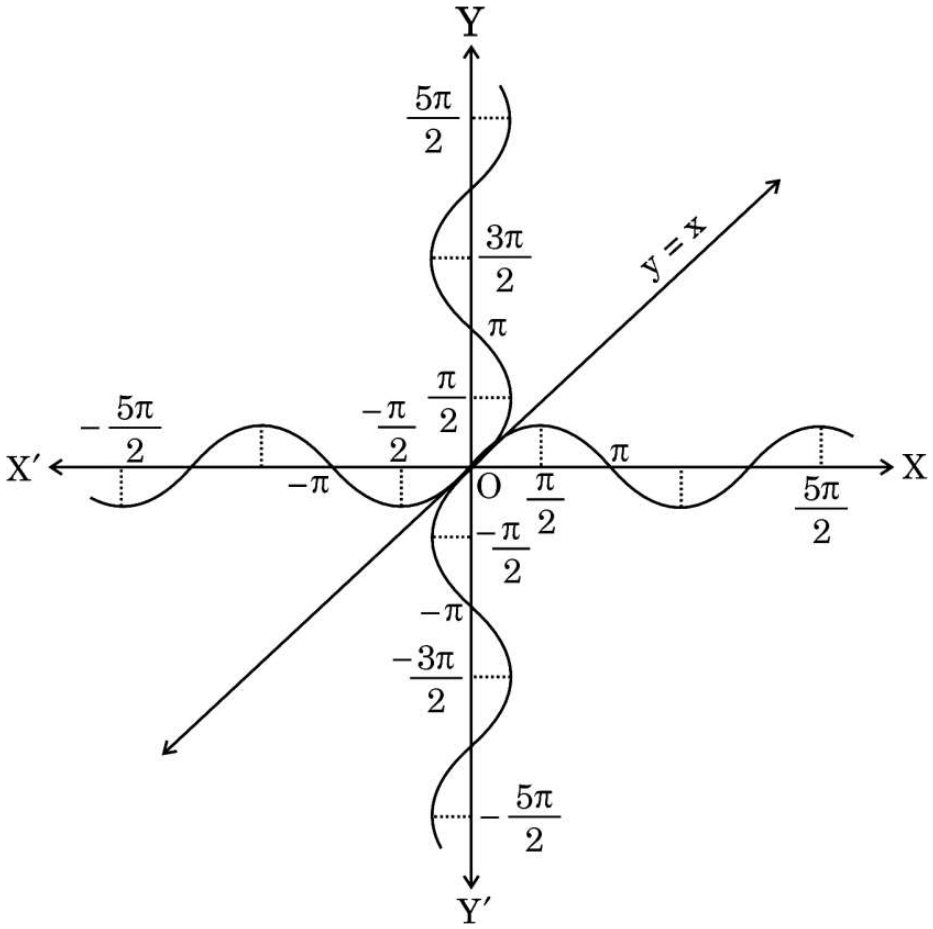
(C) $m \times m$ (D) $m \times n$

2. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A is a/an :

(A) scalar matrix (B) identity matrix

(C) symmetric matrix (D) skew-symmetric matrix

3. निम्नलिखित आलेख बना है :



(A) $y = \sin^{-1} x$ और $y = \cos^{-1} x$ से

(B) $y = \cos^{-1} x$ और $y = \cos x$ से

(C) $y = \sin^{-1} x$ और $y = \sin x$ से

(D) $y = \cos^{-1} x$ और $y = \sin x$ से

4. समान कोटि के दो विषम-सममित आव्यूहों का योगफल सदैव :

(A) एक विषम-सममित आव्यूह होता है

(B) एक सममित आव्यूह होता है

(C) एक शून्य आव्यूह होता है

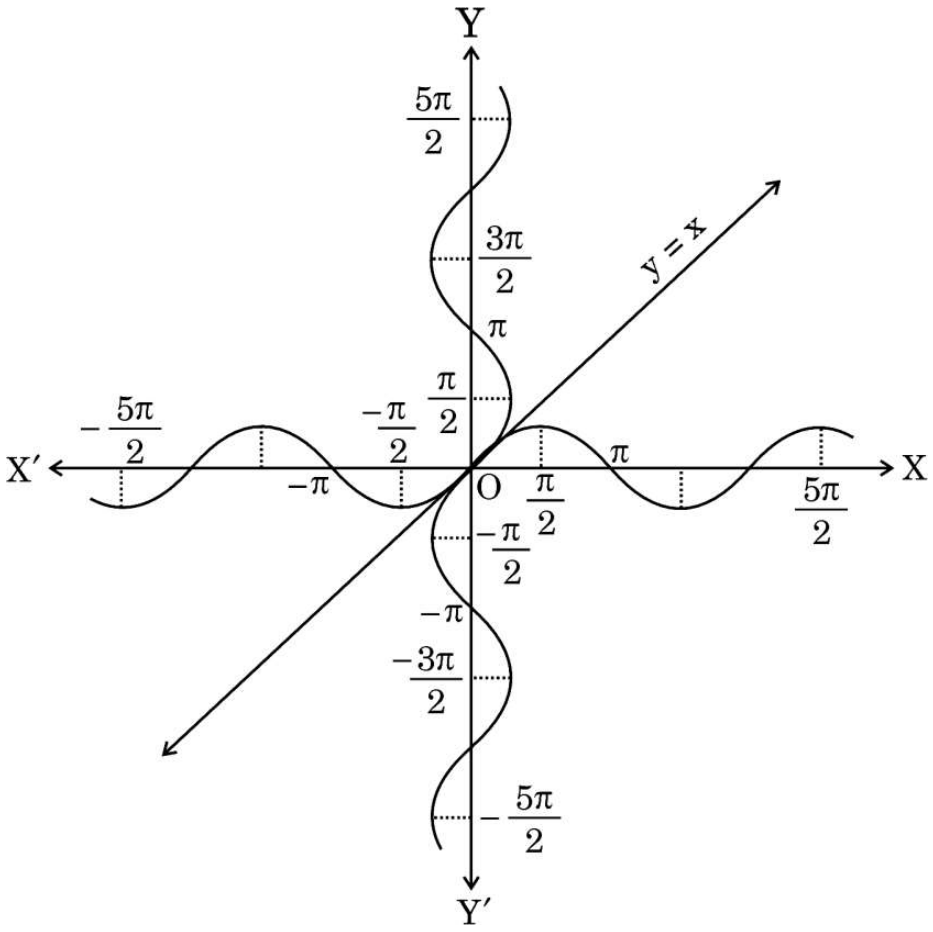
(D) एक तत्समक आव्यूह होता है

5. $\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ बराबर है :

(A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$

(C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

3. The following graph is a combination of :



(A) $y = \sin^{-1} x$ and $y = \cos^{-1} x$

(B) $y = \cos^{-1} x$ and $y = \cos x$

(C) $y = \sin^{-1} x$ and $y = \sin x$

(D) $y = \cos^{-1} x$ and $y = \sin x$

4. Sum of two skew-symmetric matrices of same order is always a/an :

(A) skew-symmetric matrix

(B) symmetric matrix

(C) null matrix

(D) identity matrix

5. $\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to :

(A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$

(C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

6. यदि $f(x) = \begin{cases} \frac{\log(1+ax)+\log(1-bx)}{x}, & x \neq 0 \text{ के लिए} \\ k, & x = 0 \text{ के लिए} \end{cases}$

$x = 0$ पर संतत है, तो k का मान है :

(A) a (B) a + b

(C) a – b (D) b

7. यदि $\tan^{-1}(x^2 - y^2) = a$, जहाँ 'a' एक अचर है, तो $\frac{dy}{dx}$ है :

(A) $\frac{x}{y}$ (B) $-\frac{x}{y}$

(C) $\frac{a}{x}$ (D) $\frac{a}{y}$

8. यदि $y = a\cos(\log x) + b\sin(\log x)$ है, तो $x^2y_2 + xy_1$ है :

(A) cot (log x) (B) y

(C) – y (D) tan (log x)

9. माना $f(x) = |x|, x \in R$. तब, निम्नलिखित में से कौन-सा कथन ***ग़लत*** है ?

(A) f का न्यूनतम मान x = 0 पर है।

(B) R में f का कोई अधिकतम मान नहीं है।

(C) x = 0 पर f संतत है।

(D) x = 0 पर f अवकलनीय है।

10. माना $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, $f(1) = 0$ है। तो, f(x) है :

(A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$ (B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$

(C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$

11. $\int \frac{x+5}{(x+6)^2} e^x \, dx$ बराबर है :

(A) log (x + 6) + C (B) $e^x + C$

(C) $\frac{e^x}{x+6} + C$ (D) $\frac{-1}{(x+6)^2} + C$

6. If $f(x) = \begin{cases} \dfrac{\log(1+ax)+\log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$

is continuous at x = 0, then the value of k is :

(A) a (B) a + b

(C) a – b (D) b

7. If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\frac{dy}{dx}$ is :

(A) $\frac{x}{y}$ (B) $-\frac{x}{y}$

(C) $\frac{a}{x}$ (D) $\frac{a}{y}$

8. If $y = a\cos(\log x) + b\sin(\log x)$, then $x^2y_2 + xy_1$ is :

(A) cot (log x) (B) y

(C) – y (D) tan (log x)

9. Let $f(x) = |x|$, $x \in R$. Then, which of the following statements is ***incorrect*** ?

(A) f has a minimum value at x = 0.

(B) f has no maximum value in R.

(C) f is continuous at x = 0.

(D) f is differentiable at x = 0.

10. Let $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, f(1) = 0. Then, f(x) is :

(A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$ (B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$

(C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$

11. $\int \frac{x+5}{(x+6)^2} e^x \, dx$ is equal to :

(A) log (x + 6) + C (B) $e^x + C$

(C) $\frac{e^x}{x+6} + C$ (D) $\frac{-1}{(x+6)^2} + C$

12. निम्नलिखित अवकल समीकरण की कोटि और घात क्रमश: हैं :

$$-\frac{d^4y}{dx^4} + 2e^{dy/dx} + y^2 = 0$$

(A) $-4, 1$ (B) 4, परिभाषित नहीं

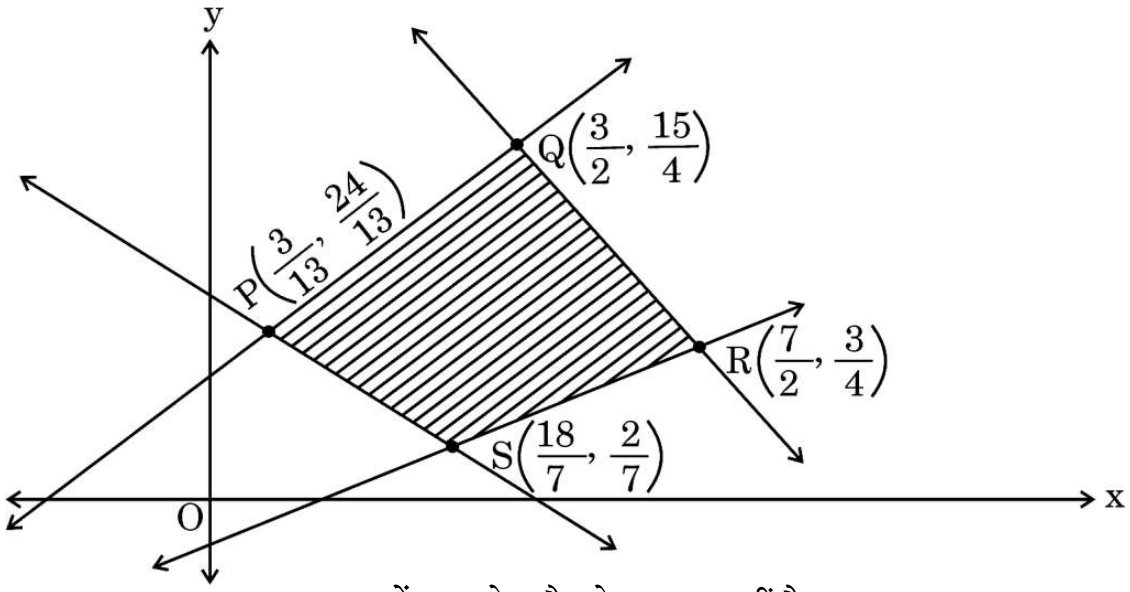
(C) 1, 1 (D) 4, 1

13. अवकल समीकरण $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ का हल है :

(A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$

(C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$

14. एक रैखिक प्रोग्रामन समस्या (LPP) के लिए, दिया गया उद्देश्य फलन $Z = x + 2y$ है। व्यवरोधों से बनने वाला सुसंगत क्षेत्र PQRS छायांकित क्षेत्र द्वारा आलेख में दिखाया गया है।



(ध्यान दें : आलेख पैमाने अनुसार नहीं है)

$$P \equiv \left(\frac{3}{13}, \frac{24}{13}\right),\ Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right),\ R \equiv \left(\frac{7}{2}, \frac{3}{4}\right),\ S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$$

निम्नलिखित में से कौन-सा कथन सही है ?

(A) Z का न्यूनतम मान $S\left(\frac{18}{7}, \frac{2}{7}\right)$ पर है

(B) Z का अधिकतम मान $R\left(\frac{7}{2}, \frac{3}{4}\right)$ पर है

(C) (Z का मान P पर) > (Z का मान Q पर)

(D) (Z का मान Q पर) < (Z का मान R पर)

12. The order and degree of the following differential equation are, respectively :

$$-\frac{d^4y}{dx^4} + 2e^{dy/dx} + y^2 = 0$$

(A) $-4, 1$ (B) 4, not defined

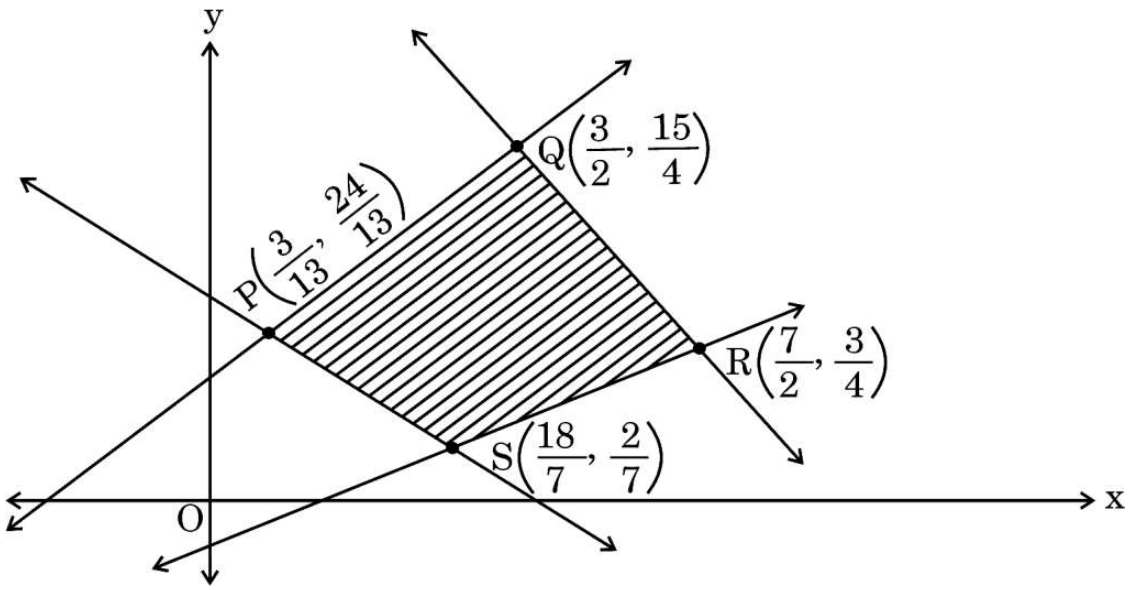
(C) 1, 1 (D) 4, 1

13. The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is :

(A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$

(C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$

14. For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.



(Note : The figure is not to scale)

$$P \equiv \left(\frac{3}{13}, \frac{24}{13}\right), Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right), R \equiv \left(\frac{7}{2}, \frac{3}{4}\right), S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$$

Which of the following statements is correct ?

(A) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$

(B) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$

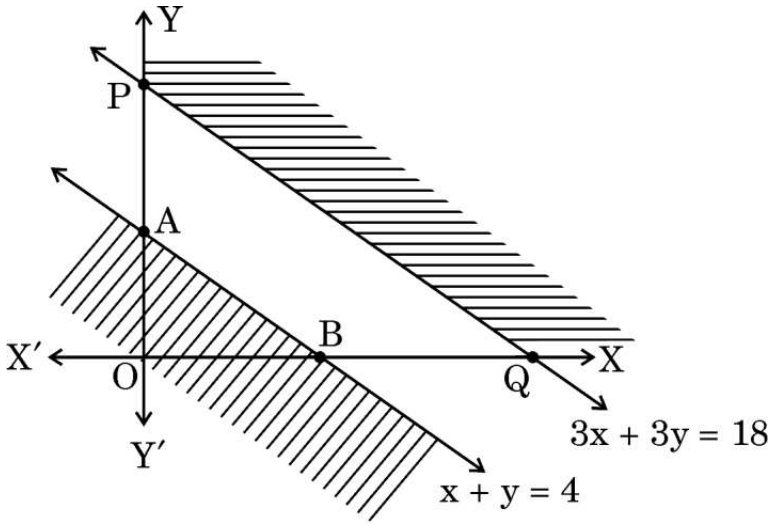
(C) (Value of Z at P) > (Value of Z at Q)

(D) (Value of Z at Q) < (Value of Z at R)

15. एक रैखिक प्रोग्रामन समस्या (LPP) के लिए, उद्देश्य फलन $Z = 2x + 5y$ का निम्नलिखित व्यवरोधों के अंतर्गत अधिकतमीकरण करना है :

$$x + y \leq 4, \quad 3x + 3y \geq 18, \quad x, y \geq 0$$

आलेख का अध्ययन कीजिए और निम्नलिखित से सही विकल्प चुनिए।



(ध्यान दें : आलेख पैमाने अनुसार नहीं है)

दी गई रैखिक प्रोग्रामन समस्या का हल :

(A) छायांकित अपरिबद्ध क्षेत्र में है।

(B) त्रिभुज AOB में है।

(C) नहीं है।

(D) त्रिभुज AOB और छायांकित अपरिबद्ध क्षेत्र के इकट्ठे क्षेत्र में है।

16. माना $|\vec{a}| = 5$ और $-2 \leq \lambda \leq 1$ है। तो, $|\lambda \vec{a}|$ का परिसर है :

(A) [5, 10] (B) [– 2, 5]

(C) [– 2, 1] (D) [–10, 5]

17. वक्र $y^2 = x$, का $x = 0$ तथा $x = 1$ के बीच परिबद्ध क्षेत्र का क्षेत्रफल है :

(A) $\frac{3}{2}$ वर्ग इकाई (B) $\frac{2}{3}$ वर्ग इकाई

(C) 3 वर्ग इकाई (D) $\frac{4}{3}$ वर्ग इकाई

18. एक डिब्बे में 4 हरे, 8 नीले और 3 लाल पेन हैं। एक छात्र इस डिब्बे से यादृच्छया एक पेन निकालता है और इसके रंग को जानने के पश्चात वापस डिब्बे में रख देता है। वह इस प्रक्रिया को 3 बार दोहराता है। कम-से-कम एक बार लाल पेन आने की प्रायिकता है :

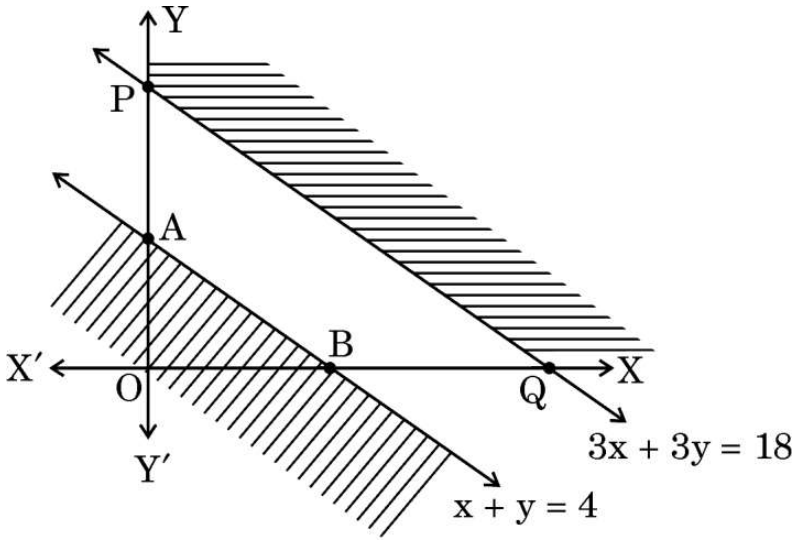
(A) $\frac{124}{125}$ (B) $\frac{1}{125}$

(C) $\frac{61}{125}$ (D) $\frac{64}{125}$

15. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints :

$x + y \leq 4,\ 3x + 3y \geq 18,\ x, y \geq 0$

Study the graph and select the correct option.



(Note : The figure is not to scale)

The solution of the given LPP :

(A) lies in the shaded unbounded region.

(B) lies in Δ AOB.

(C) does not exist.

(D) lies in the combined region of Δ AOB and unbounded shaded region.

16. Let $|\vec{a}| = 5$ and $-2 \leq \lambda \leq 1$. Then, the range of $|\lambda \vec{a}|$ is :

(A) [5, 10] (B) [– 2, 5]

(C) [– 2, 1] (D) [–10, 5]

17. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is :

(A) $\frac{3}{2}$ sq units (B) $\frac{2}{3}$ sq units

(C) 3 sq units (D) $\frac{4}{3}$ sq units

18. A box has 4 green, 8 blue and 3 red pens. A student picks up a pen at random, checks its colour and replaces it in the box. He repeats this process 3 times. The probability that at least one pen picked was red is :

(A) $\frac{124}{125}$ (B) $\frac{1}{125}$

(C) $\frac{61}{125}$ (D) $\frac{64}{125}$

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं । दो कथन दिए गए हैं, जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है । इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए ।*

(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है ।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है ।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है ।

(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है ।

19. *अभिकथन (A) :* यदि $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 256$ तथा $|\vec{b}| = 8$ है, तो $|\vec{a}| = 2$ है ।

तर्क (R) : $\sin^2\theta + \cos^2\theta = 1$ तथा $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$ और $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$ है ।

20. *अभिकथन (A) :* माना $f(x) = e^x$ तथा $g(x) = \log x$ है । तो $(f + g)\, x = e^x + \log x$ है, जहाँ $(f + g)$ का प्रांत R है ।

तर्क (R) : $(f + g)$ का प्रांत = (f) का प्रांत $\cap$ (g) का प्रांत ।

खण्ड ख

इस खण्ड में 5 अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं ।

21. $f(x) = \sin^{-1}(-x^2)$ का प्रांत ज्ञात कीजिए ।

22. (क) $e^{\sqrt{2x}}$ के सापेक्ष $\sqrt{e^{\sqrt{2x}}}$ का अवकलज ज्ञात कीजिए, $x > 0$ के लिए ।

अथवा

(ख) यदि $(x)^y = (y)^x$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए ।

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 256$ and $|\vec{b}| = 8$, then $|\vec{a}| = 2$.

Reason (R) : $\sin^2\theta + \cos^2\theta = 1$ and

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$ and $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$.

20. *Assertion (A) :* Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)\, x = e^x + \log x$ where domain of $(f + g)$ is R.

Reason (R) : $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Find the domain of $f(x) = \sin^{-1}(-x^2)$.

22. (a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$.

OR

(b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.

23. x के मान ज्ञात कीजिए जिनके लिए $f(x) = \frac{x-4}{x+1}$, $x \neq -1$ एक वर्धमान अथवा ह्रासमान फलन है।

24. (क) यदि बिंदुओं A और B के क्रमश: $\vec{a}$ और $\vec{b}$ स्थिति सदिश हों, तो बिंदु C, जो बढ़ाए गए BA पर स्थित है, का स्थिति सदिश ज्ञात कीजिए जबकि BC = 3BA हो।

अथवा

(ख) सदिश $\vec{r}$ तीन अक्षों x, y और z के साथ समान कोण पर झुका हुआ है। यदि $\vec{r}$ का परिमाण $5\sqrt{3}$ इकाई है, तो $\vec{r}$ ज्ञात कीजिए।

25. ज्ञात कीजिए कि क्या रेखाएँ $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ तथा $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ परस्पर प्रतिच्छेदी हैं।

खण्ड ग

इस खण्ड में 6 लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं।

26. माना $A = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ तथा $C = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$ दो आव्यूह हैं। तो, आव्यूह B ज्ञात कीजिए, जिससे AB = C हो।

27. (क) x के सापेक्ष $y = \sin^{-1}(3x - 4x^3)$ का अवकलज ज्ञात कीजिए, यदि $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ है।

अथवा

(ख) x के सापेक्ष $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ का अवकलज ज्ञात कीजिए, जब $x \in (0, 1)$ है।

23. Determine the values of x for which $f(x) = \frac{x-4}{x+1}$, $x \neq -1$ is an increasing or a decreasing function.

24. (a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA.

OR

(b) Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.

25. Determine if the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect with each other.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. Let $A = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$ be two matrices. Then, find the matrix B if AB = C.

27. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

OR

(b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.

28. (क) एक छात्र प्राकृत संख्याओं के युग्म (जोड़े) इस प्रकार बनाना चाहता है कि वे समीकरण $2x + y = 41$ को संतुष्ट करते हों, $x, y \in N$। इस तरह बने संबंध का प्रांत और परिसर ज्ञात कीजिए। जाँच कीजिए कि क्या यह संबंध स्वतुल्य, सममित अथवा संक्रामक है। अत: ज्ञात कीजिए कि क्या यह एक तुल्यता संबंध है अथवा नहीं।

अथवा

(ख) दर्शाइए कि $f(n) = \begin{cases} n-1, & \text{यदि } n \text{ सम है} \\ n+1, & \text{यदि } n \text{ विषम है} \end{cases}$

द्वारा प्रदत्त फलन $f : N \to N$, जहाँ N प्राकृत संख्याओं का समुच्चय है, एकैकी आच्छादी है।

29. रैखिक प्रोग्रामन समस्या पर विचार कीजिए, जहाँ उद्देश्य फलन $Z = (x + 4y)$ का निम्न व्यवरोधों

$$2x + y \geq 1000$$

$$x + 2y \geq 800$$

$$x, y \geq 0$$

के अंतर्गत न्यूनतमीकरण करना है।

सुसंगत क्षेत्र का एक स्वच्छ आलेख बनाइए और Z का न्यूनतम मान ज्ञात कीजिए।

30. (क) बिंदु $P(2, 4, -1)$ की रेखा $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ से दूरी ज्ञात कीजिए।

अथवा

(ख) माना बिंदुओं A, B और C के स्थिति सदिश क्रमश: $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ और $\hat{i} + 5\hat{j} + 3\hat{k}$ हैं। बिंदु A से गुज़रने वाली और BC के समांतर रेखा के सदिश और कार्तीय समीकरण ज्ञात कीजिए।

31. एक व्यक्ति दो स्वतंत्र चयन समितियों I और II का प्रमुख है। यदि उसकी समिति I में ग़लत चयन की प्रायिकता 0·03 तथा समिति II में ग़लत चयन की प्रायिकता 0·01 है, तो प्रायिकता ज्ञात कीजिए कि इस व्यक्ति ने :

(i) दोनों समितियों में सही चयन किया है।

(ii) केवल एक ही समिति में सही चयन किया है।

28. (a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

OR

(b) Show that the function $f : N \to N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, & \text{if n is even} \\ n+1, & \text{if n is odd} \end{cases}$ is a bijection.

29. Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints

$2x + y \geq 1000$

$x + 2y \geq 800$

$x, y \geq 0$.

Draw a neat graph of the feasible region and find the minimum value of Z.

30. (a) Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

OR

(b) Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.

31. A person is Head of two independent selection committees I and II. If the probability of making a wrong selection in committee I is 0·03 and that in committee II is 0·01, then find the probability that the person makes the correct decision of selection :

(i) in both committees

(ii) in only one committee

खण्ड घ

इस खण्ड में 4 दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं।

32. (क) ज्ञात कीजिए :

$$\int \frac{x^2 + 1}{(x-1)^2 (x+3)} \, dx$$

अथवा

(ख) मान ज्ञात कीजिए :

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x} \, dx$$

33. वक्र $y = 2 + |x + 1|$ का रफ आलेख बनाइए और समाकलन के प्रयोग से वक्र $y = 2 + |x + 1|$, $x = -4$, $x = 3$ और $y = 0$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

34. (क) अवकल समीकरण $x^2y \, dx - (x^3 + y^3) \, dy = 0$ का हल ज्ञात कीजिए।

अथवा

(ख) अवकल समीकरण $(1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0$ की प्रारंभिक स्थिति $y(0) = 0$ के अंतर्गत हल ज्ञात कीजिए।

35. मान लीजिए कि दर्पण के पॉलिश किए हुए भाग, रेखा $\frac{x}{1} = \frac{1-y}{-2} = \frac{2z-4}{6}$ के अनुदिश है। दर्पण से कुछ दूरी पर एक बिंदु P(1, 6, 3) का प्रतिबिम्ब दर्पण के पीछे बनता है। प्रतिबिम्ब बिंदु के निर्देशांक तथा बिंदु P और उसके प्रतिबिम्ब के बीच की दूरी ज्ञात कीजिए।

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find :

$$\int \frac{x^2 + 1}{(x-1)^2 (x+3)}\, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\, dx$$

33. Draw a rough sketch for the curve $y = 2 + |x + 1|$. Using integration, find the area of the region bounded by the curve $y = 2 + |x + 1|$, $x = -4$, $x = 3$ and $y = 0$.

34. (a) Solve the differential equation : $x^2y\, dx - (x^3 + y^3)\, dy = 0$.

OR

(b) Solve the differential equation $(1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.

35. Let the polished side of the mirror be along the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{2z-4}{6}$.

A point P(1, 6, 3), some distance away from the mirror, has its image formed behind the mirror. Find the coordinates of the image point and the distance between the point P and its image.

खण्ड ङ

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं।

प्रकरण अध्ययन – 1

36. तीन विद्यार्थी नेहा, रानी और सैम स्टेशनरी का सामान खरीदने के लिए बाज़ार जाते हैं। नेहा ने 4 पेन, 3 कापियाँ और 2 रबड़, ₹ 60 देकर खरीदे। रानी ने 2 पेन, 4 कापियाँ और 6 रबड़, ₹ 90 देकर खरीदे। सैम ने ₹ 70 में 6 पेन, 2 कापियाँ और 3 रबड़ खरीदे।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) प्रत्येक वस्तु का मूल्य ज्ञात करने की समस्या को हल करने के लिए आवश्यक समीकरण बनाइए और इनको आव्यूह रूप $AX = B$ में व्यक्त कीजिए। *1*

(ii) $|A|$ ज्ञात कीजिए और पुष्टि कीजिए कि क्या A^{-1} ज्ञात करना संभव है। *1*

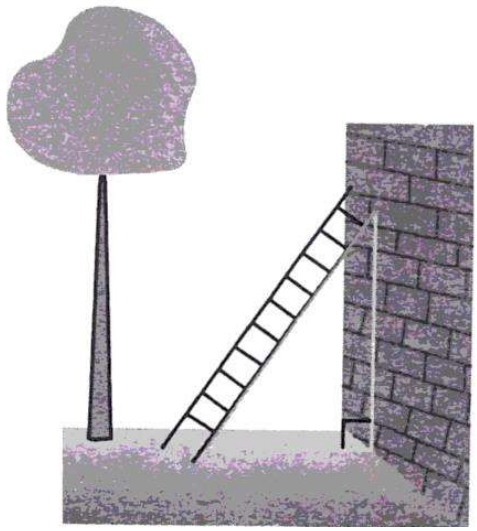
(iii) (क) यदि संभव है, तो A^{-1} ज्ञात कीजिए और X ज्ञात करने के लिए सूत्र लिखिए। *2*

अथवा

(iii) (ख) $A^2 - 8I$, जहाँ I एक तत्समक आव्यूह है, ज्ञात कीजिए। *2*

प्रकरण अध्ययन – 2

37.



निश्चित लंबाई 'h' की एक सीढ़ी दीवार के साथ इस प्रकार लगाई जाती है कि वह दीवार की ऊँचाई के साथ चलने के लिए स्वतंत्र है।

उपर्युक्त सूचना के आधार पर, निम्नलिखित प्रश्नों के उत्तर दीजिए :

(i) एक निश्चित क्षण में दीवार और सीढ़ी के निचले हिस्से के बीच की दूरी (y) को 'h' और दीवार पर ऊँचाई (x) के रूप में व्यक्त कीजिए। इसके अलावा, एक पर्यवेक्षक द्वारा किनारे से देखे गए समकोण त्रिभुज के क्षेत्रफल (A) के लिए एक अभिव्यक्ति, h तथा x के पदों में लिखिए। *1*

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.

Based upon the above information, answer the following questions :

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B. *1*

(ii) Find |A| and confirm if it is possible to find A^{-1}. *1*

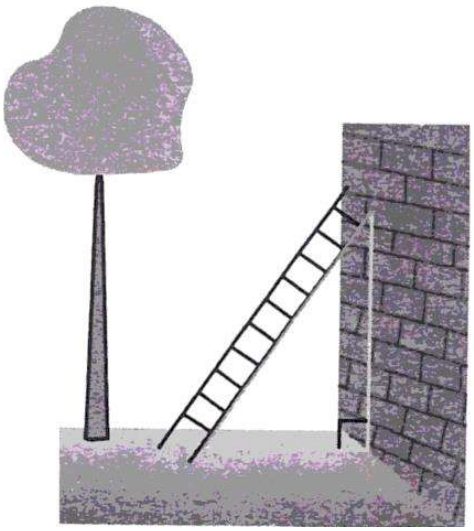
(iii) (a) Find A^{-1}, if possible, and write the formula to find X. *2*

OR

(iii) (b) Find $A^2 - 8I$, where I is an identity matrix. *2*

Case Study – 2

37.



A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions :

(i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. *1*

(ii) दीवार पर ऊँचाई (x) के सापेक्ष, क्षेत्रफल (A) का अवकलज ज्ञात कीजिए और इसका क्रांतिक बिंदु भी ज्ञात कीजिए। *1*

(iii) (क) दर्शाइए कि क्रांतिक बिंदु पर, समकोण त्रिभुज का क्षेत्रफल (A) अधिकतम होगा। *2*

अथवा

(iii) (ख) यदि 5 m लम्बी सीढ़ी का पाद, दीवार की ओर इस प्रकार खींचा जा रहा है कि दूरी (y) घटने की दर 2 m/s है, तो दीवार पर ऊँचाई (x) किस दर से बढ़ रही है, जब सीढ़ी का पाद दीवार से 3 m दूर है ? *2*

प्रकरण अध्ययन – 3

38. इलेक्ट्रॉनिक सामान बेचने वाली एक दुकान, केवल तीन प्रतिष्ठित कंपनियों A, B और C के स्मार्टफोन बेचती है क्योंकि उनके दोषपूर्ण स्मार्टफोन बनाने की प्रायिकताएँ केवल क्रमश: 5%, 4% और 2% हैं। उसकी सूची में, उसके पास कंपनी A के 25% स्मार्टफोन, कंपनी B के 35% स्मार्टफोन और कंपनी C के 40% स्मार्टफोन हैं।

एक व्यक्ति इस दुकान से एक स्मार्टफोन खरीदता है।

(i) इस फोन के दोषपूर्ण होने की प्रायिकता ज्ञात कीजिए। *2*

(ii) यह दोषपूर्ण स्मार्टफोन, कंपनी B के द्वारा उत्पादित किए जाने की प्रायिकता क्या है ? *2*

(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point. *1*

(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. *2*

OR

(iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ? *2*

Case Study – 3

38. A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

A person buys a smartphone from this shop.

(i) Find the probability that it was defective. *2*

(ii) What is the probability that this defective smartphone was manufactured by company B ? *2*

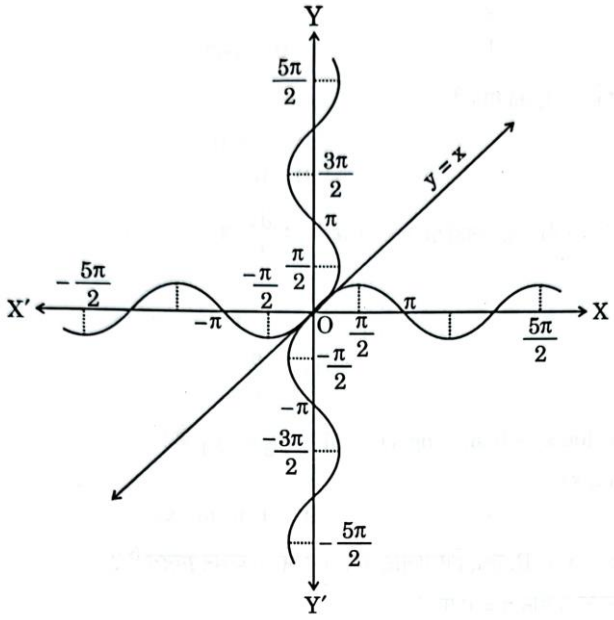
Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary Examination, 2025
SUBJECT NAME- MATHEMATICS (Q.P. CODE – 65/6/1)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.

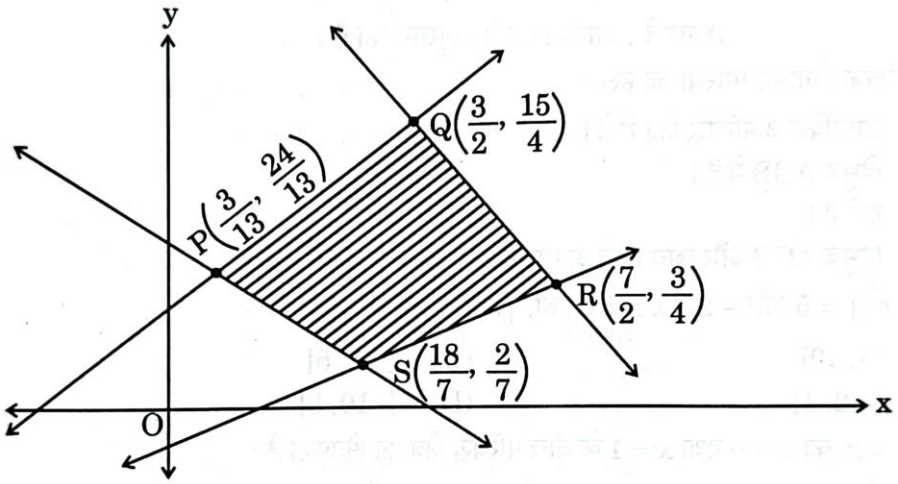
10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks_____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of the answer marked correct and the rest as wrong, but no marks
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

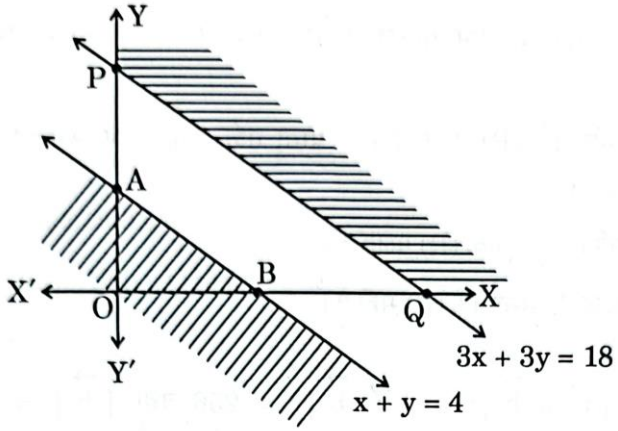
MARKING SCHEME – 65/6/1

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A This section comprises multiple choice questions (MCQs) of 1 mark each.	
1.	Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is : (A) $n \times n$ (B) $n \times m$ (C) $m \times m$ (D) $m \times n$	
Ans	(B) $n \times m$	1
2.	If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A is a/an : (A) scalar matrix (B) identity matrix (C) symmetric matrix (D) skew-symmetric matrix	
Ans	(C) symmetric matrix	1
3.	The following graph is a combination of :  (A) $y = \sin^{-1} x$ and $y = \cos^{-1} x$ (B) $y = \cos^{-1} x$ and $y = \cos x$ (C) $y = \sin^{-1} x$ and $y = \sin x$ (D) $y = \cos^{-1} x$ and $y = \sin x$	
Ans	(C) $y = \sin^{-1}x$ and $y = \sin x$	1
4.	Sum of two skew-symmetric matrices of same order is always a/an : (A) skew-symmetric matrix (B) symmetric matrix (C) null matrix (D) identity matrix	

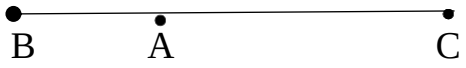
Ans	(A) skew-symmetric matrix	1
5.	$\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to : (A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$ (C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$	
Ans	(D) $\frac{7\pi}{12}$	1
6.	If $f(x) = \begin{cases} \frac{\log(1+ax)+\log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at x = 0, then the value of k is : (A) a (B) a + b (C) a – b (D) b	
Ans	(C) a – b	1
7.	If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\frac{dy}{dx}$ is : (A) $\frac{x}{y}$ (B) $-\frac{x}{y}$ (C) $\frac{a}{x}$ (D) $\frac{a}{y}$	
Ans	(A) $\frac{x}{y}$	1
8.	If $y = a\cos(\log x) + b\sin(\log x)$, then $x^2y_2 + xy_1$ is : (A) cot (log x) (B) y (C) – y (D) tan (log x)	
Ans	(C) –y	1
9.	Let f(x) = |x|, x ∈ R. Then, which of the following statements is ***incorrect*** ? (A) f has a minimum value at x = 0. (B) f has no maximum value in R. (C) f is continuous at x = 0. (D) f is differentiable at x = 0.	
Ans	(D) f is differentiable at x = 0	1
10.	Let $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, f(1) = 0. Then, f(x) is : (A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$ (B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$ (C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$	
Ans	(B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$	1

11.	$\int \frac{x+5}{(x+6)^2} e^x \, dx$ is equal to : (A) $\log(x+6) + C$ (B) $e^x + C$ (C) $\frac{e^x}{x+6} + C$ (D) $\frac{-1}{(x+6)^2} + C$	
Ans	(C) $\frac{e^x}{x+6} + C$	1
12.	The order and degree of the following differential equation are, respectively : $-\frac{d^4y}{dx^4} + 2e^{dy/dx} + y^2 = 0$ (A) $-4, 1$ (B) 4, not defined (C) 1, 1 (D) 4, 1	
Ans	(B) 4, not defined	1
13.	The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is : (A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$ (C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$	
Ans	(D) $3e^{-4y} + 4e^{3x} + 12C = 0$	1

14.	For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.  (Note : The figure is not to scale) $P \equiv \left(\frac{3}{13}, \frac{24}{13}\right), Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right), R \equiv \left(\frac{7}{2}, \frac{3}{4}\right), S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$ Which of the following statements is correct ? (A) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$ (B) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$ (C) (Value of Z at P) > (Value of Z at Q) (D) (Value of Z at Q) < (Value of Z at R)	
Ans	(A) Z is minimum at S $(\frac{18}{7}, \frac{2}{7})$	1

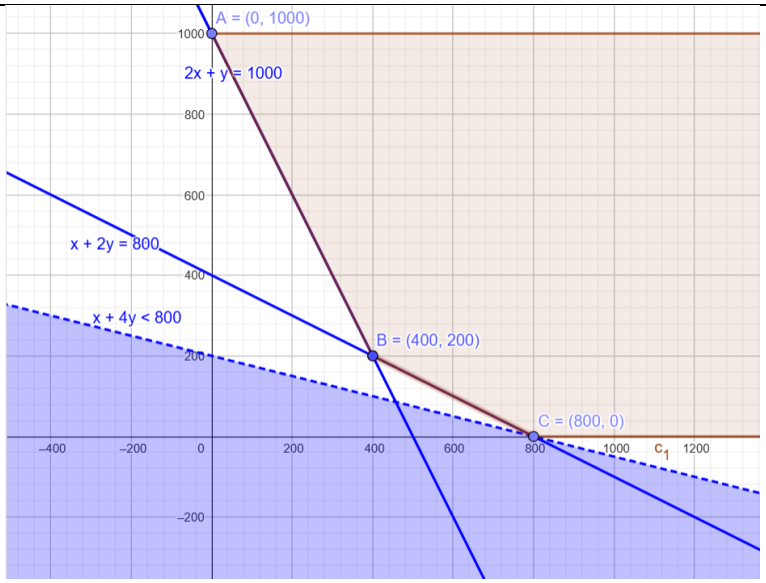
15.	In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints : $x + y \leq 4,\ 3x + 3y \geq 18,\ x, y \geq 0$ Study the graph and select the correct option.  (Note : The figure is not to scale) The solution of the given LPP : (A) lies in the shaded unbounded region. (B) lies in Δ AOB. (C) does not exist. (D) lies in the combined region of Δ AOB and unbounded shaded region.	
Ans	(C) does not exist	1
16.	Let $\lvert \vec{a} \rvert = 5$ and $-2 \leq \lambda \leq 1$. Then, the range of $\lvert \lambda \vec{a} \rvert$ is : (A) [5, 10] (B) [– 2, 5] (C) [– 2, 1] (D) [–10, 5]	
Ans	1 mark for any attempt as correct answer is not given in any option	1
17.	The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is : (A) $\frac{3}{2}$ sq units (B) $\frac{2}{3}$ sq units (C) 3 sq units (D) $\frac{4}{3}$ sq units	
Ans	(D) $\frac{4}{3}$ sq units	1
18.	A box has 4 green, 8 blue and 3 red pens. A student picks up a pen at random, checks its colour and replaces it in the box. He repeats this process 3 times. The probability that at least one pen picked was red is : (A) $\frac{124}{125}$ (B) $\frac{1}{125}$ (C) $\frac{61}{125}$ (D) $\frac{64}{125}$	
Ans	(C) $\frac{61}{125}$	1

	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* If $\lvert \vec{a} \times \vec{b} \rvert^2 + \lvert \vec{a} . \vec{b} \rvert^2 = 256$ and $\lvert \vec{b} \rvert = 8$, then $\lvert \vec{a} \rvert = 2$. *Reason (R) :* $\sin^2\theta + \cos^2\theta = 1$ and $\lvert \vec{a} \times \vec{b} \rvert = \lvert \vec{a} \rvert \lvert \vec{b} \rvert \sin\theta$ and $\vec{a} . \vec{b} = \lvert \vec{a} \rvert \lvert \vec{b} \rvert \cos\theta$.	
Ans	(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).	1
20.	*Assertion (A) :* Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)\, x = e^x + \log x$ where domain of $(f + g)$ is R. *Reason (R) :* $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.	
Ans	(D) Assertion (A) is false, but Reason (R) is true.	1
	SECTION-B This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.	
21.	**Find the domain of $f(x) = \sin^{-1}(-x^2)$.**	
Ans	(a) $-1 \le -x^2 \le 1 \Rightarrow -1 \le -x^2 \le 0$ $\Rightarrow 0 \le x^2 \le 1 \Rightarrow -1 \le x \le 1$	1 1
22.	**(a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$.** **OR** **(b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.**	
Ans	(a) Let $u = \sqrt{e^{\sqrt{2x}}}$ and $v = e^{\sqrt{2x}}$ Derivative of $\sqrt{v}$ w.r.t. $v = \frac{1}{2\sqrt{v}}$	½ 1

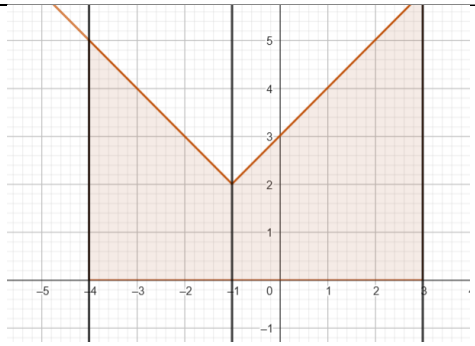
	Required derivative = $\frac{1}{2\sqrt{e^{\sqrt{2x}}}}$.	½
	OR	
	Taking log on both sides, we get y logx = x logy	½
	Differentiating both sides w.r.t. x, we get	
	$\frac{y}{x.} + \log x\frac{dy}{dx} = \frac{x}{y.}\frac{dy}{dx} + \log y$	1
	$\Rightarrow \frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$	½
23.	**Determine the values of x for which $f(x) = \frac{x-4}{x+1}$, $x \neq -1$ is an increasing or a decreasing function.**	
Ans	$f'(x) = \frac{x+1-x+4}{(x+1)^2} = \frac{5}{(x+1)^2} > 0$	$1\frac{1}{2}$
	Hence f is increasing in its domain.	½
24.	**(a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA.**	
	OR	
	(b) Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.	
Ans	(a) C divides BA in the ratio 3 : 2 externally	1
	Required vector = $\vec{c} = \frac{3\vec{a}-2\vec{b}}{3-2} = 3\vec{a} - 2\vec{b}$ 	1
	OR	
	(b) Unit vector equally inclined along coordinate axes is $\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$	1
	$\vec{r} = 5\sqrt{3}(\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}) = 5\hat{i} + 5\hat{j} + 5\hat{k}$ (or $-5\hat{i} - 5\hat{j} - 5\hat{k}$)	1
25.	**Determine if the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect with each other.**	
Ans	$\vec{b}_1 = 3\hat{i} - \hat{j}$, $\vec{b}_2 = 2\hat{i} + 3\hat{k}$, $\vec{a}_2 = 4\hat{i} - \hat{k}$, $\vec{a}_1 = \hat{i} + \hat{j} - \hat{k}$	
	$(\vec{a}_2 - \vec{a}_1) = 3\hat{i} - \hat{j}$	½
	$\vec{b}_1 \times \vec{b}_2 = -3\hat{i} - 9\hat{j} + 2\hat{k}$	1

	$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = (3\hat{i} - \hat{j}).(-3\hat{i} - 9\hat{j} + 2\hat{k}) = 0$, hence lines intersect	½
	SECTION-C This section comprises 6 Short Answer (SA) type questions of 3 marks each.	
26.	***is section comprises 6 Short Answer (SA) type questions of 3 marks each.*** **Let $A = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$ be two matrices. Then, find the matrix B if AB = C.**	
Ans	(a) Let B = [x y z] $AB = C \implies \begin{bmatrix} x & y & z \\ 4x & 4y & 4z \\ -2x & -2y & -2z \end{bmatrix} = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$ which gives x = 3, y = 4 and z = 2 B = [3 4 2]	½ 2 ½
27.	**27. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.** **OR** **(b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.**	
Ans	(a) $x = \sin t$ gives $y = \sin^{-1}(\sin 3t) = 3t = 3\sin^{-1}x$ $\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$ Aliter: $\frac{dy}{dx} = \frac{3-12x^2}{\sqrt{1-(3x-4x^3)^2}}$ OR (b) $x = \tan t$ gives $y = \cos^{-1}(\cos 2t) = 2t = 2\tan^{-1}x$ $\frac{dy}{dx} = \frac{2}{1+x^2}$ Aliter: $\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-4x}{(1+x^2)^2}$.	$\frac{1}{2}+1+\frac{1}{2}$ 1 3 $\frac{1}{2}+1+\frac{1}{2}$ 1 3

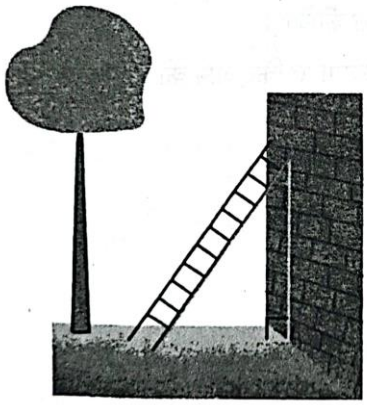
28.	(a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not. **OR** (b) Show that the function $f : N \to N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, \text{ if } n \text{ is even} \\ n+1, \text{ if } n \text{ is odd} \end{cases}$ is a bijection.	
Ans	(a) R = {(1,39), (2, 37),_ _ _ ,(20, 1)} Domain = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20} Range ={1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39} (1, 1) does not belong to R hence not reflexive (1, 39) belongs to R but (39, 1) does not belong to R hence not symmetric (11, 19) and (19, 3) belong to R but (11, 3) does not belong to R hence not transitive. Hence R is not an equivalence relation. OR (a) Let f(x) = f(y) Let x and y are both odd or both even Then either x+1 = y + 1 or x-1 = y-1 gives x = y x odd and y even is rejected as x + 1 = y -1 gives x – y = -2 not possible as odd number and even number cannot differ by 2 Hence f is one-one For onto: Let f(x) = y gives x = y + 1 or x = y - 1 If y is odd, x is even and if y is even, x is odd. Range = N = co-domain, hence onto As f is both one-one and onto hence bijective	½ ½ ½ ½ 1 ½ ½ ½ 1 ½
29.	Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints $2x + y \geq 1000$ $x + 2y \geq 800$ $x, y \geq 0.$ Draw a neat graph of the feasible region and find the minimum value of Z.	

Ans	 Correct Graph and shading: Corner points Value of Z (800, 0) 800 (400, 200) 1200 (0, 1000) 4000 $x + 4y < 800$ has no region common with feasible region, hence 800 is minimum	1 ½ 1 ½
30.	(a) Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. **OR** (b) Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.	
Ans	(a) Let $\vec{a_2} = 2\hat{i} + 4\hat{j} - \hat{k}$, $\vec{a_1} = -5\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$ Distance between point and line is given by $d = \frac{\lvert(\vec{a_2} - \vec{a_1}) \times \vec{b}\rvert}{\lvert\vec{b}\rvert}$ Here $(\vec{a_2} - \vec{a_1}) = 7\hat{i} + 7\hat{j} - 7\hat{k}$ $(\vec{a_2} - \vec{a_1}) \times \vec{b} = -35\hat{i} + 56\hat{j} + 21\hat{k}$ $d = \frac{49\sqrt{2}}{7\sqrt{2}} = 7$ OR (b) Vector parallel to given line = $3\hat{j} + 4\hat{k}$	½ $1\frac{1}{2}$ 1 1

	Vector equation is $\vec{r} = 3\hat{i} - \hat{j} - 2\hat{k} + \mu(3\hat{j} + 4\hat{k})$ Cartesian equation is $\frac{x-3}{0} = \frac{y+1}{3} = \frac{z+2}{4}$	1 1
31.	**31.** **A person is Head of two independent selection committees I and II. If the probability of making a wrong selection in committee I is 0·03 and that in committee II is 0·01, then find the probability that the person makes the correct decision of selection :** **(i) in both committees** **(ii) in only one committee**	
Ans	(i) P(correct decision in both committees) =(1- 0.03) . (1- 0.01) = 0.9603 (ii) P(correct decision in one committee) = 0.03 . (1- 0.01) + (1- 0.03). 0.01 = 0.0394	$1+\frac{1}{2}$ 1 $\frac{1}{2}$
	SECTION-D This section comprises 4 Long Answer (LA) type questions of 5 marks each.	
32.	(a) **Find :** $\int \frac{x^2+1}{(x-1)^2(x+3)}\,dx$ **OR** (b) **Evaluate :** $\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\,dx$	
Ans	(a) $\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3} = \frac{3/8}{x-1} + \frac{1/2}{(x-1)^2} + \frac{5/8}{x+3}$ $I = \frac{3}{8}\log\|x-1\| - \frac{1}{2(x-1)} + \frac{5}{8}\log\|x+3\| + C$ OR (b) Let $I = \int_0^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x}\,dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}-x}{\cos x+\sin x}\,dx$ using property $2I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}}{\sin x+\cos x}\,dx$ $= \frac{\pi}{2}\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}} \frac{1}{\sin(\frac{\pi}{4}+x)}\,dx$ $2I = \frac{\pi}{2\sqrt{2}}\log\left\|\operatorname{cosec}\left(\frac{\pi}{4}+x\right) - \cot\left(\frac{\pi}{4}+x\right)\right\|_0^{\frac{\pi}{2}}$	1 + 2 2 1 1 1 1

	$I=\frac{\pi}{4\sqrt{2}}log\frac{\sqrt{2}+1}{\sqrt{2}-1}$	1
33.	**Draw a rough sketch for the curve $y = 2 + |x + 1|$. Using integration, find the area of the region bounded by the curve $y = 2 + |x + 1|$, $x = -4$, $x = 3$ and $y = 0$.**	
Ans	 Correct Graph: Required area $= \int_{-4}^{-1}(2 + |x + 1|)\,dx + \int_{-1}^{3}(2 + |x + 1|)\,dx$ $= \int_{-4}^{-1}(1 - x)\,dx + \int_{-1}^{3}(3 + x)\,dx$ $= -\frac{(1-x)^2}{2}|_{-4}^{-1} + \frac{(3+x)^2}{2}|_{-1}^{3}$ $= \frac{21}{2} + 16 = \frac{53}{2}$	1 1 1 1 1
34.	**34. (a) Solve the differential equation : $x^2y\,dx - (x^3 + y^3)\,dy = 0$.** **OR** **(b) Solve the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.**	
Ans	(a) Given differential equation can be written as $\frac{dy}{dx} = \frac{yx^2}{x^3+y^3}$ Put $y = vx$, so $\frac{dv}{dx} = v + x\frac{dv}{dx}$ Therefore, $v + x\frac{dv}{dx} = \frac{vx^3}{x^3+v^3x^3} = \frac{v}{1+v^3}$ $x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$ $(\frac{1}{v^4} + \frac{1}{v})dv = \frac{-dx}{x}$ Integrating we get $\frac{-1}{3v^3} + \log|v| = -\log|x| + C$	½ 1 1 1 1

	$\frac{-x^3}{3y^3}$+ log$\lvert y\rvert$ = C	½
	OR	
	(b) Given D.E. is $\frac{dy}{dx} + \frac{2x}{1+x^2}\, y = \frac{4x^2}{1+x^2}$	1
	Integrating factor is $e^{\int \frac{2x}{1+x^2}dx} = e^{log\,(1+x^2)} = (1+x^2)$	1
	Solution is $y(1+x^2) = \int 4x^2 dx + C$	1
	$y(1+x^2) = \frac{4x^3}{3} + C$	1
	y(0) = 0 gives C = 0, hence solution is $y(1+x^2) = \frac{4x^3}{3}$	1
35.	Let the polished side of the mirror be along the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{2z-4}{6}$. A point P(1, 6, 3), some distance away from the mirror, has its image formed behind the mirror. Find the coordinates of the image point and the distance between the point P and its image.	
Ans	Equation of given line is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$	1
	Let coordinates of point on the line be (λ , 2λ + 1, 3λ + 2) for some λ	½
	Drs of line perpendicular to line along mirror are <λ-1, 2λ - 5, 3λ - 1>	½
	(λ-1).1 +(2λ - 5).2+ (3λ - 1).3 = 0 gives λ = 1	1
	Coordinates of foot of perpendicular are (1,3,5)	1
	For image	
	$\frac{x+1}{2} = 1$, $\frac{y+6}{2} = 3$, $\frac{z+3}{2} = 5$ gives image as (1, 0, 7)	½
	Required distance = $\sqrt{0+36+16} = 2\sqrt{13}$	½
	SECTION-E This section comprises 3 case study-based questions of 4 marks each	
36.	**Case Study – 1** Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers. Based upon the above information, answer the following questions : (i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B. *1* (ii) Find $\lvert A\rvert$ and confirm if it is possible to find A^{-1}. *1* (iii) (a) Find A^{-1}, if possible, and write the formula to find X. *2* **OR** (iii) (b) Find $A^2 - 8I$, where I is an identity matrix. *2*	

Ans	(i)Let the price of each pen, notepad, eraser be ₹x, ₹y and ₹z respectively Given system in the form AX = B is $\begin{pmatrix}4 & 3 & 2\\ 2 & 4 & 6\\ 6 & 2 & 3\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}60\\ 90\\ 70\end{pmatrix}$	1
	(ii)$\lvert A\rvert = 50 \neq 0$, hence A^{-1}exists	1
	(iii) (a) $A^{-1}=\frac{\text{adj}A}{\lvert A\rvert} = \frac{1}{50}\begin{pmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{pmatrix}$ $X = A^{-1}B$	1 ½ ½
	OR (iii)(b) $A^2 = \begin{pmatrix}4 & 3 & 2\\ 2 & 4 & 6\\ 6 & 2 & 3\end{pmatrix}\begin{pmatrix}4 & 3 & 2\\ 2 & 4 & 6\\ 6 & 2 & 3\end{pmatrix} = \begin{pmatrix}34 & 28 & 32\\ 52 & 34 & 46\\ 46 & 32 & 33\end{pmatrix}$ $A^2 - 8I = \begin{pmatrix}26 & 28 & 32\\ 52 & 26 & 46\\ 46 & 32 & 25\end{pmatrix}$	1 ½ ½
37.	**Case Study – 2**  A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall. Based upon the above information, answer the following questions : (i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. *1*	

	(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point. *1* (iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. *2* **OR** (iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ? *2*	
Ans	(i) $y^2 = h^2 - x^2$ $A = \frac{1}{2}xy = \frac{1}{2}x\sqrt{h^2 - x^2}$ ii) $\frac{dA}{dx} = \frac{1}{2}\sqrt{h^2 - x^2} + \frac{1}{2}x\frac{-x}{\sqrt{h^2 - x^2}}$ $\frac{dA}{dx} = 0$ gives $x = \frac{h}{\sqrt{2}}$ (iii)(a) $A'' = \frac{1}{2}\frac{-4x.\sqrt{h^2 - x^2} - (h^2 - 2x^2)\frac{-x}{\sqrt{h^2 - x^2}}}{h^2 - x^2}$ is < 0 at $x = \frac{h}{\sqrt{2}}$ Hence A is maximum at critical point OR (iii)(b) $y^2 = 25 - x^2$ hence $y = 3$ gives $x = 4$ $2y\frac{dy}{dt} = -2x\frac{dx}{dt}$ $\frac{dx}{dt} = 1.5$m/s	½ ½ ½ ½ $1\frac{1}{2}$ ½ ½ 1 ½

38.	A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C. A person buys a smartphone from this shop. (i) Find the probability that it was defective. 2 (ii) What is the probability that this defective smartphone was manufactured by company B ? 2	
Ans	(i) P(defective smartphone) = $0.25\times0.05 + 0.35\times 0.04 + 0.40\times0.02$ $= 0.0345$ (ii) P(B/ Defective) = $\frac{0.35\times 0.04}{0.25\times 0.05 + 0.35\times0.04 + 0.40\times0.02}$ $= \frac{140}{345}$ or $\frac{28}{69}$	1 ½ ½ 1 ½ ½

Series PQ2RS/2

Set – 1

प्रश्न-पत्र कोड
Q.P. Code 65/2/1

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ* ***23*** *हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न-पत्र में* ***38*** *प्रश्न हैं ।*
- *प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains* ***23*** *printed pages.*
- *Please check that this question paper contains* ***38*** *questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे *अधिकतम अंक : 80*

Time allowed : 3 hours *Maximum Marks : 80*

सामान्य निर्देश :

निम्नलिखित निर्देशों को बहुत सावधानी से पढ़िए और उनका सख़्ती से पालन कीजिए :

(i) *इस प्रश्न-पत्र में **38** प्रश्न हैं । **सभी** प्रश्न **अनिवार्य** हैं ।*

(ii) *यह प्रश्न-पत्र **पाँच** खण्डों में विभाजित है – **क, ख, ग, घ** एवं **ङ** ।*

(iii) ***खण्ड क** में प्रश्न संख्या **1** से **18** तक बहुविकल्पीय तथा प्रश्न संख्या **19** एवं **20** अभिकथन एवं तर्क आधारित **1** अंक के प्रश्न हैं ।*

(iv) ***खण्ड ख** में प्रश्न संख्या **21** से **25** तक अति लघु-उत्तरीय (VSA) प्रकार के **2** अंकों के प्रश्न हैं ।*

(v) ***खण्ड ग** में प्रश्न संख्या **26** से **31** तक लघु-उत्तरीय (SA) प्रकार के **3** अंकों के प्रश्न हैं ।*

(vi) ***खण्ड घ** में प्रश्न संख्या **32** से **35** तक दीर्घ-उत्तरीय (LA) प्रकार के **5** अंकों के प्रश्न हैं ।*

(vii) ***खण्ड ङ** में प्रश्न संख्या **36** से **38** प्रकरण अध्ययन आधारित **4** अंकों के प्रश्न हैं ।*

(viii) *प्रश्न-पत्र में समग्र विकल्प नहीं दिया गया है । यद्यपि, खण्ड ख के 2 प्रश्नों में, खण्ड ग के 3 प्रश्नों में, खण्ड घ के 2 प्रश्नों में तथा खण्ड ङ के 2 प्रश्नों में आंतरिक विकल्प का प्रावधान दिया गया है ।*

(ix) *कैल्कुलेटर का उपयोग **वर्जित** है ।*

खण्ड क

इस खण्ड में बहुविकल्पीय प्रश्न हैं, जिनमें प्रत्येक प्रश्न 1 अंक का है ।

1. यदि एक 3×3 के अदिश आव्यूह के सभी अवयवों का योगफल 9 है, तो इसके सभी अवयवों का गुणनफल होगा :

(A) 0 (B) 9

(C) 27 (D) 729

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is :

(A) 0 (B) 9

(C) 27 (D) 729

2. माना $f : R_+ \to [-5, \infty)$, $f(x) = 9x^2 + 6x - 5$ द्वारा परिभाषित है, जहाँ R_+ सभी ऋणेतर वास्तविक संख्याओं का समुच्चय है । तो f है :

(A) एकैकी

(B) आच्छादक

(C) एकैकी आच्छादक

(D) न तो एकैकी और न ही आच्छादक

3. यदि $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$ है, तो k का मान है :

(A) 0 (B) 1

(C) 2 (D) 4

4. $f(x) = \begin{cases} |x|+3, & \text{यदि } x \le -3 \\ -2x, & \text{यदि } -3 < x < 3 \\ 6x+2, & \text{यदि } x \ge 3 \end{cases}$

द्वारा परिभाषित फलन के असांतत्यता के बिंदुओं की संख्या है :

(A) 0 (B) 1

(C) 2 (D) अनन्त

5. फलन $f(x) = x^3 - 3x^2 + 12x - 18$:

(A) R पर निरंतर ह्रासमान है

(B) R पर निरंतर वर्धमान है

(C) R पर न तो निरंतर वर्धमान है और न ही निरंतर ह्रासमान है

(D) $(-\infty, 0)$ पर निरंतर ह्रासमान है

2. Let $f : R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is :

(A) one-one

(B) onto

(C) bijective

(D) neither one-one nor onto

3. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :

(A) 0 (B) 1

(C) 2 (D) 4

4. The number of points of discontinuity of $f(x) = \begin{cases} |x|+3, & \text{if } x \le -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x+2, & \text{if } x \ge 3 \end{cases}$ is :

(A) 0 (B) 1

(C) 2 (D) infinite

5. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :

(A) strictly decreasing on R

(B) strictly increasing on R

(C) neither strictly increasing nor strictly decreasing on R

(D) strictly decreasing on $(-\infty, 0)$

6. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x \cos x} dx$ बराबर है :

(A) π (B) शून्य (0)

(C) $\int_0^{\pi/2} \frac{2\sin x}{1+\sin x \cos x} dx$ (D) $\frac{\pi^2}{4}$

7. अवकल समीकरण $\frac{dy}{dx} = F(x, y)$ समघातीय अवकल समीकरण नहीं होगा, यदि $F(x, y)$ है :

(A) $\cos x - \sin\left(\frac{y}{x}\right)$ (B) $\frac{y}{x}$

(C) $\frac{x^2+y^2}{xy}$ (D) $\cos^2\left(\frac{x}{y}\right)$

8. किन्हीं दो सदिशों $\vec{a}$ और $\vec{b}$ के लिए, निम्नलिखित कथनों में से कौन-सा सदैव सही है ?

(A) $\vec{a} \cdot \vec{b} \geq |\vec{a}||\vec{b}|$ (B) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$

(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$ (D) $\vec{a} \cdot \vec{b} < |\vec{a}||\vec{b}|$

9. बिन्दु (0, 1, 2) से x-अक्ष पर डाले गए लम्ब के पाद के निर्देशांक हैं :

(A) (1, 0, 0) (B) (2, 0, 0)

(C) $(\sqrt{5}, 0, 0)$ (D) (0, 0, 0)

10. एक रैखिक प्रोग्रामन समस्या के सभी व्यवरोधों द्वारा नियत उभयनिष्ठ क्षेत्र कहलाता है :

(A) एक अपरिबद्ध क्षेत्र (B) एक इष्टतम क्षेत्र

(C) एक परिबद्ध क्षेत्र (D) एक सुसंगत क्षेत्र

6. $\int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x \cos x}\,dx$ is equal to :

(A) π (B) Zero (0)

(C) $\int_0^{\pi/2} \frac{2\sin x}{1+\sin x \cos x}\,dx$ (D) $\frac{\pi^2}{4}$

7. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if F(x, y) is :

(A) $\cos x - \sin\left(\frac{y}{x}\right)$ (B) $\frac{y}{x}$

(C) $\frac{x^2+y^2}{xy}$ (D) $\cos^2\left(\frac{x}{y}\right)$

8. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ?

(A) $\vec{a} \cdot \vec{b} \geq |\vec{a}||\vec{b}|$ (B) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$

(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$ (D) $\vec{a} \cdot \vec{b} < |\vec{a}||\vec{b}|$

9. The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by :

(A) (1, 0, 0) (B) (2, 0, 0)

(C) $(\sqrt{5}, 0, 0)$ (D) (0, 0, 0)

10. The common region determined by all the constraints of a linear programming problem is called :

(A) an unbounded region (B) an optimal region

(C) a bounded region (D) a feasible region

11. माना किसी प्रयोग में E किसी प्रतिदर्श समष्टि S की एक घटना है, तो $P(S|E)$ बराबर है :

(A) $P(S \cap E)$ (B) $P(E)$

(C) 1 (D) 0

12. यदि $A = [a_{ij}]$ एक 3×3 आव्यूह है, जिसमें $a_{ij} = i - 3j$ है, तो निम्नलिखित में से कौन-सा ***गलत*** है ?

(A) $a_{11} < 0$ (B) $a_{12} + a_{21} = -6$

(C) $a_{13} > a_{31}$ (D) $a_{31} = 0$

13. x के सापेक्ष, $\tan^{-1}(x^2)$ का अवकलज है :

(A) $\frac{x}{1+x^4}$ (B) $\frac{2x}{1+x^4}$

(C) $-\frac{2x}{1+x^4}$ (D) $\frac{1}{1+x^4}$

14. अवकल समीकरण $(y'')^2 + (y')^3 = x \sin(y')$ की घात :

(A) 1 है (B) 2 है

(C) 3 है (D) परिभाषित नहीं है

15. मात्रक सदिश, जो कि सदिशों $\hat{i} + \hat{k}$ और $\hat{i} - \hat{k}$, दोनों पर लंब है, है :

(A) $2\hat{j}$ (B) $\hat{j}$

(C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$

16. रेखा $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ के समांतर एक सदिश के दिक्-अनुपात हैं :

(A) 2, – 1, 6 (B) 2, 1, 6

(C) 2, 1, 3 (D) 2, – 1, 3

11. Let E be an event of a sample space S of an experiment, then $P(S \mid E)$ =

(A) $P(S \cap E)$ (B) $P(E)$

(C) 1 (D) 0

12. If $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = i - 3j$, then which of the following is ***false*** ?

(A) $a_{11} < 0$ (B) $a_{12} + a_{21} = -6$

(C) $a_{13} > a_{31}$ (D) $a_{31} = 0$

13. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is :

(A) $\frac{x}{1+x^4}$ (B) $\frac{2x}{1+x^4}$

(C) $-\frac{2x}{1+x^4}$ (D) $\frac{1}{1+x^4}$

14. The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')$ is :

(A) 1 (B) 2

(C) 3 (D) not defined

15. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is :

(A) $2\hat{j}$ (B) $\hat{j}$

(C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$

16. Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :

(A) 2, – 1, 6 (B) 2, 1, 6

(C) 2, 1, 3 (D) 2, – 1, 3

17. यदि $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ तथा $[F(x)]^2 = F(kx)$ है, तो k का मान है :

(A) 1 (B) 2

(C) 0 (D) – 2

18. यदि एक रेखा x-अक्ष की धनात्मक दिशा के साथ 30° का कोण, y-अक्ष की धनात्मक दिशा के साथ 120° का कोण बनाती है, तो यह रेखा z-अक्ष की धनात्मक दिशा के साथ जो कोण बनाती है, वह है :

(A) 90° (B) 120°

(C) 60° (D) 0°

*प्रश्न संख्या **19** और **20** अभिकथन एवं तर्क आधारित प्रश्न हैं । दो कथन दिए गए हैं जिनमें एक को अभिकथन (A) तथा दूसरे को तर्क (R) द्वारा अंकित किया गया है । इन प्रश्नों के सही उत्तर नीचे दिए गए कोडों (A), (B), (C) और (D) में से चुनकर दीजिए ।*

(A) अभिकथन (A) और तर्क (R) दोनों सही हैं और तर्क (R), अभिकथन (A) की सही व्याख्या करता है ।

(B) अभिकथन (A) और तर्क (R) दोनों सही हैं, परन्तु तर्क (R), अभिकथन (A) की सही व्याख्या ***नहीं*** करता है ।

(C) अभिकथन (A) सही है, परन्तु तर्क (R) ग़लत है ।

(D) अभिकथन (A) ग़लत है, परन्तु तर्क (R) सही है ।

19. *अभिकथन (A) :* किसी एक सममित आव्यूह A के लिए, B′AB एक विषम सममित आव्यूह होता है ।

तर्क (R) : एक वर्ग आव्यूह P विषम सममित आव्यूह कहलाएगा, यदि $P' = -P$.

20. *अभिकथन (A) :* दो शून्येतर सदिशों $\vec{a}$ और $\vec{b}$ के लिए, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

तर्क (R) : दो शून्येतर सदिशों $\vec{a}$ और $\vec{b}$ के लिए, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

17. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is :

(A) 1 (B) 2

(C) 0 (D) – 2

18. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is :

(A) 90° (B) 120°

(C) 60° (D) 0°

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* For any symmetric matrix A, B′AB is a skew-symmetric matrix.

Reason (R) : A square matrix P is skew-symmetric if $P' = -P$.

20. *Assertion (A) :* For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

Reason (R) : For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

खण्ड ख

इस खण्ड में अति लघु-उत्तरीय (VSA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 2 अंक हैं ।

21. (क) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$ का मान ज्ञात कीजिए ।

अथवा

(ख) फलन $f(x) = \sin^{-1}(x^2 - 4)$ का प्रांत ज्ञात कीजिए । इसका परिसर भी ज्ञात कीजिए ।

22. (क) यदि $f(x) = |\tan 2x|$ है, तो $x = \frac{\pi}{3}$ पर $f'(x)$ का मान ज्ञात कीजिए ।

अथवा

(ख) यदि $y = \text{cosec}(\cot^{-1} x)$ है, तो सिद्ध कीजिए कि $\sqrt{1 + x^2}\,\frac{dy}{dx} - x = 0$.

23. यदि फलन $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ के स्थानीय उच्चतम और स्थानीय निम्नतम मान, क्रमश: M और m द्वारा प्रदत्त हैं, तो $(M - m)$ का मान ज्ञात कीजिए ।

24. ज्ञात कीजिए :

$$\int \frac{e^{4x} - 1}{e^{4x} + 1}\,dx$$

25. दर्शाइए कि $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ अपने प्रांत में निरंतर वर्धमान है ।

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

OR

(b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

22. (a) If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

OR

(b) If $y = \text{cosec}(\cot^{-1} x)$, then prove that $\sqrt{1 + x^2}\,\frac{dy}{dx} - x = 0$.

23. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ respectively, find the value of $(M - m)$.

24. Find :

$$\int \frac{e^{4x} - 1}{e^{4x} + 1}\,dx$$

25. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

खण्ड ग

इस खण्ड में लघु-उत्तरीय (SA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 3 अंक हैं ।

26. (क) यदि $x = e^{\cos 3t}$ और $y = e^{\sin 3t}$ है, तो सिद्ध कीजिए कि $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.

अथवा

(ख) दर्शाइए कि :

$$\frac{d}{dx}(|x|) = \frac{x}{|x|}, \; x \neq 0$$

27. (क) मान ज्ञात कीजिए :

$$\int_{-2}^{2} \sqrt{\frac{2-x}{2+x}}\, dx$$

अथवा

(ख) ज्ञात कीजिए :

$$\int \frac{1}{x\,[(\log x)^2 - 3 \log x - 4]}\, dx$$

28. (क) अवकल समीकरण $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$ का विशिष्ट हल ज्ञात कीजिए; दिया गया है $y = 2$, जब $x = 1$ है ।

अथवा

(ख) अवकल समीकरण $y\, dx = (x + 2y^2)\, dy$ का व्यापक हल ज्ञात कीजिए ।

29. त्रिभुज ABC के शीर्षों के स्थिति सदिश $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ और $C(3\hat{i} - 4\hat{j} - 4\hat{k})$ हैं । त्रिभुज ABC के सभी कोण ज्ञात कीजिए ।

30. पासों के एक जोड़े को एक साथ उछाला जाता है । यदि X, पासों के ऊपरी फलकों पर आई संख्याओं के पूर्ण-अंतर को व्यक्त करता है, तो X का प्रायिकता बंटन ज्ञात कीजिए ।

31. ज्ञात कीजिए :

$$\int x^2 \cdot \sin^{-1}(x^{3/2})\, dx$$

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.

OR

(b) Show that :

$$\frac{d}{dx}(|x|) = \frac{x}{|x|}, \; x \neq 0$$

27. (a) Evaluate :

$$\int_{-2}^{2} \sqrt{\frac{2-x}{2+x}}\, dx$$

OR

(b) Find :

$$\int \frac{1}{x\,[(\log x)^2 - 3 \log x - 4]}\, dx$$

28. (a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$; $y = 2$, when $x = 1$.

OR

(b) Find the general solution of the differential equation :

$$y\, dx = (x + 2y^2)\, dy$$

29. The position vectors of vertices of Δ ABC are $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ and $C(3\hat{i} - 4\hat{j} - 4\hat{k})$. Find all the angles of Δ ABC.

30. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.

31. Find :

$$\int x^2 \cdot \sin^{-1}(x^{3/2})\, dx$$

खण्ड घ

इस खण्ड में दीर्घ-उत्तरीय (LA) प्रकार के प्रश्न हैं, जिनमें प्रत्येक के 5 अंक हैं ।

32. (क) दर्शाइए कि फलन $f : R \to R$ जो $f(x) = \frac{2x}{1+x^2}$ द्वारा परिभाषित है, न तो एकैकी है और न ही आच्छादक है । साथ ही, समुच्चय A ज्ञात कीजिए जिससे दिया गया फलन $f : R \to A$ एक आच्छादक फलन हो जाए ।

अथवा

(ख) $N \times N$ (जहाँ N प्राकृत संख्याओं का समुच्चय है) पर एक संबंध R निम्न प्रकार प्रदत्त है :

$$(a, b)\, R\, (c, d) \Leftrightarrow a - c = b - d$$

दर्शाइए कि R एक तुल्यता संबंध है ।

33. एक रेखा का समीकरण ज्ञात कीजिए जो बिंदुओं A(2, 3, 4) और B(4, 5, 8) को मिलाने वाले रेखा-खंड को समद्विभाजित करती हो और रेखाओं $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ और $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ के लंबवत हो ।

34. (क) आव्यूहों का प्रयोग करके, निम्न समीकरण-निकाय को हल कीजिए :

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

जहाँ $x, y, z \neq 0$

अथवा

(ख) यदि $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$ है, तो दर्शाइए कि $A' A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$.

35. यदि प्रथम चतुर्थांश में $y^2 = 4x$, $x = 1$ और x-अक्ष से घिरे क्षेत्र का क्षेत्रफल A_1 द्वारा प्रदत्त है और $y^2 = 4x$, $x = 4$ से घिरे क्षेत्र का क्षेत्रफल A_2 द्वारा प्रदत्त है, तो $A_1 : A_2$ ज्ञात कीजिए ।

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Show that a function $f : R \to R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.

OR

(b) A relation R is defined on $N \times N$ (where N is the set of natural numbers) as :

$$(a, b)\ R\ (c, d) \Leftrightarrow a - c = b - d$$

Show that R is an equivalence relation.

33. Find the equation of the line which bisects the line segment joining points A(2, 3, 4) and B(4, 5, 8) and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

34. (a) Solve the following system of equations, using matrices :

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

where $x, y, z \neq 0$

OR

(b) If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that $A' A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$.

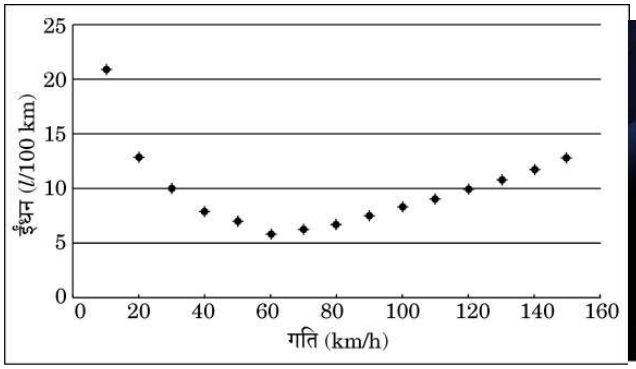
35. If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

खण्ड ङ

इस खण्ड में 3 प्रकरण अध्ययन आधारित प्रश्न हैं, जिनमें प्रत्येक के 4 अंक हैं ।

प्रकरण अध्ययन – 1

36. ओवरस्पीडिंग से ईंधन की खपत बढ़ जाती है और टायर तथा भूमि में घर्षण और वायु प्रतिरोध के परिणामस्वरूप ईंधन की बचत कम हो जाती है । जबकि वाहन अलग-अलग गति पर इष्टतम ईंधन अर्थव्यवस्था तक पहुँचते हैं, ईंधन लाभ आमतौर पर 80 km/h से ऊपर की गति पर तेज़ी से घटता है ।





कुछ बाधाओं के तहत ईंधन की खपत F (l/100 km) और गति V (km/h) के बीच संबंध $F = \frac{V^2}{500} - \frac{V}{4} + 14$ द्वारा दिया गया है ।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) F ज्ञात कीजिए, जबकि V = 40 km/h. *1*

(ii) $\frac{dF}{dV}$ ज्ञात कीजिए । *1*

(iii) (क) वह गति V ज्ञात कीजिए जब ईंधन खपत F न्यूनतम होती है । *2*

अथवा

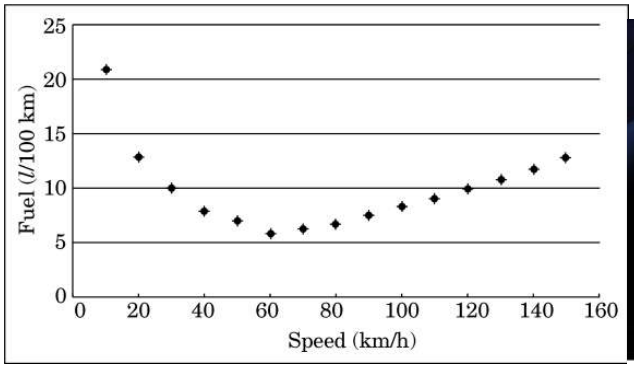
(iii) (ख) V गति से 600 km की यात्रा करने के लिए आवश्यक ईंधन की मात्रा ज्ञात कीजिए जिस पर $\frac{dF}{dV} = -0{\cdot}01$ है । *2*

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.





The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of the above information, answer the following questions :

(i) Find F, when V = 40 km/h. *1*

(ii) Find $\frac{dF}{dV}$. *1*

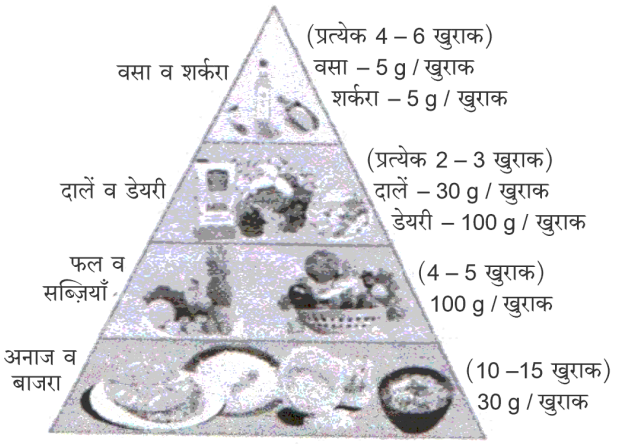
(iii) (a) Find the speed V for which fuel consumption F is minimum. *2*

OR

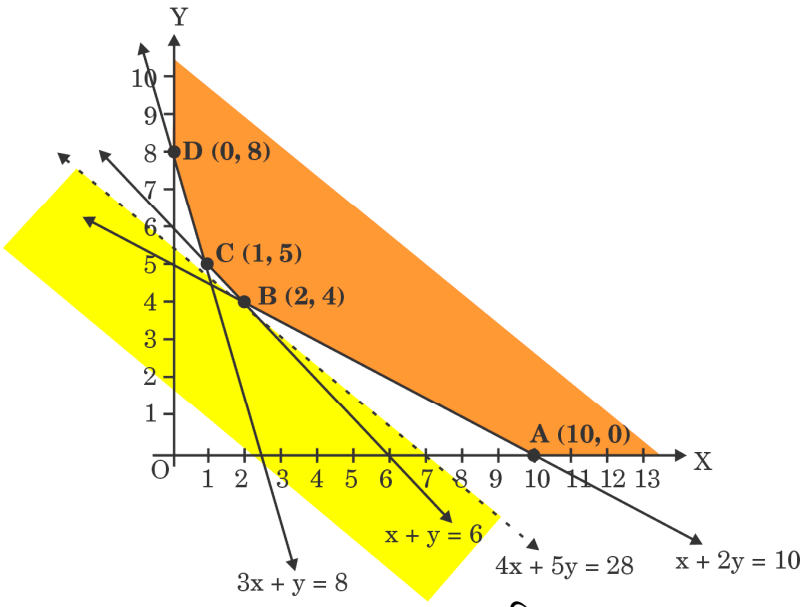
(iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0{\cdot}01$. *2*

प्रकरण अध्ययन – 2

37. सितंबर का महीना पूरे देश में राष्ट्रीय पोषण माह के रूप में मनाया जाता है । शरीर को आवश्यक उचित पोषक तत्त्व प्रदान करने के लिए स्वस्थ और संतुलित आहार का पालन करना महत्त्वपूर्ण है । संतुलित आहार हमें मानसिक रूप से भी फिट रखता है और ऊर्जा के बेहतर स्तर को बढ़ावा देता है ।



आकृति-1



आकृति-2

एक आहार विशेषज्ञ दो प्रकार के खाद्य पदार्थों, फूड X (x kg) और फूड Y (y kg), जो क्रमश: ₹ 16/kg और ₹ 20/kg की दर पर उपलब्ध हैं, से युक्त आहार की लागत को कम करना चाहता है । व्यवरोधों द्वारा नियत सुसंगत क्षेत्र आकृति-2 में दिखाया गया है ।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) आकृति-2 से उन सभी व्यवरोधों को पहचानिए और लिखिए जो दिए गए सुसंगत क्षेत्र को निर्धारित करते हैं । *2*

(ii) यदि उद्देश्य लागत फलन $Z = 16x + 20y$ को न्यूनतम करना हो, तो x और y का मान ज्ञात कीजिए जिस पर लागत न्यूनतम हो । यह मानते हुए कि दिए गए अपरिबद्ध क्षेत्र से न्यूनतम लागत संभव है, न्यूनतम लागत भी ज्ञात कीजिए । *2*

Case Study – 2

37. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

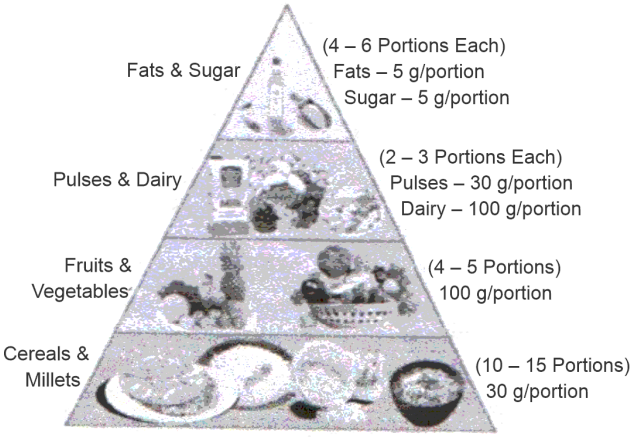


Figure-1

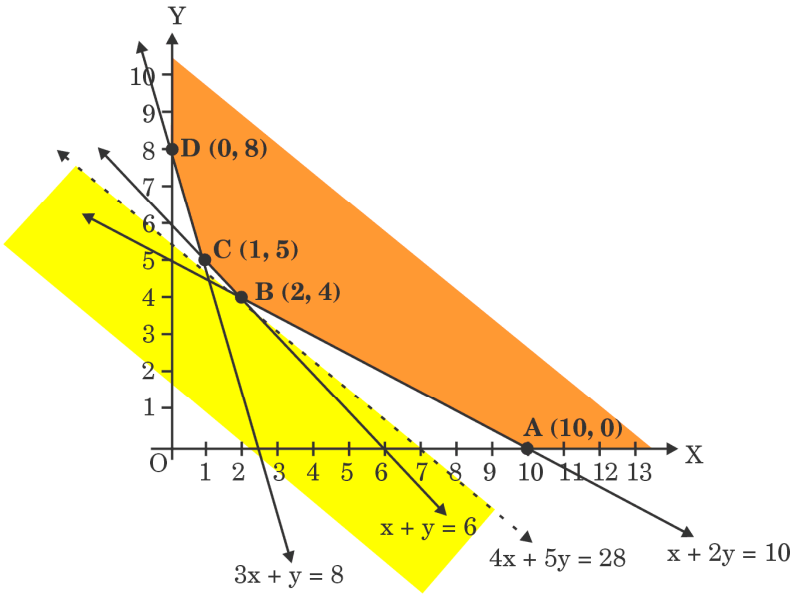


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

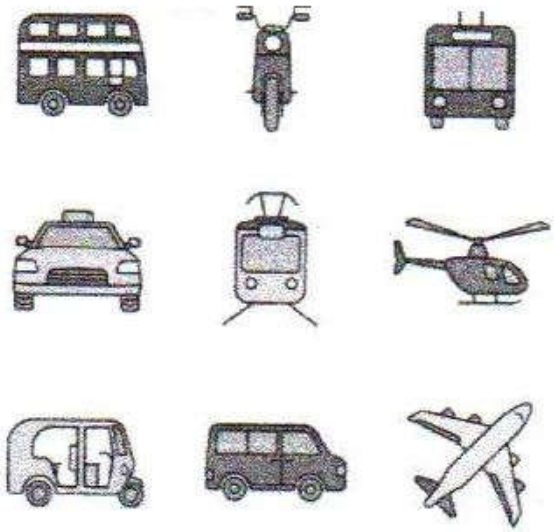
On the basis of the above information, answer the following questions :

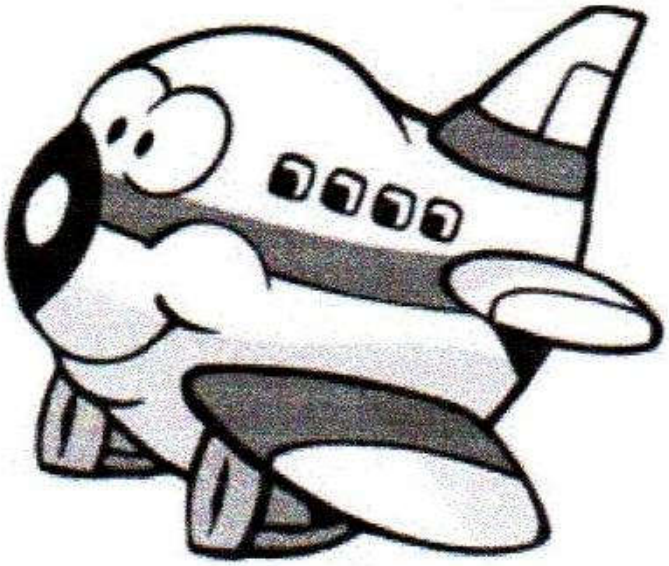
(i) Identify and write all the constraints which determine the given feasible region in Figure-2. *2*

(ii) If the objective is to minimize cost Z = 16x + 20y, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region. *2*

प्रकरण अध्ययन – 3

38. जब परिवहन किए गए यात्रियों की संख्या को व्यक्तिगत चोटों और मृत्यु के योग के आधार पर मापा जाता है, तो हवाई जहाज अब तक परिवहन का सबसे सुरक्षित साधन है ।





पिछले रिकॉर्ड बताते हैं कि हवाई जहाज के दुर्घटनाग्रस्त होने की प्रायिकता 0·00001% है । इसके अलावा, 95% संभावना है कि विमान दुर्घटना के बाद जीवित बचे लोग होंगे । मान लीजिए कि दुर्घटना न होने की स्थिति में सभी यात्री जीवित बच जाते हैं ।

मान लीजिए E_1 घटना है कि एक विमान की दुर्घटना हुई है और E_2 वह घटना है कि कोई दुर्घटना नहीं हुई है । माना A वह घटना है जिसमें यात्री यात्रा के बाद जीवित बच जाते हैं ।

उपर्युक्त सूचना के आधार पर, निम्न प्रश्नों के उत्तर दीजिए :

(i) हवाई जहाज के दुर्घटनाग्रस्त न होने की प्रायिकता ज्ञात कीजिए । *1*

(ii) $P(A \mid E_1) + P(A \mid E_2)$ ज्ञात कीजिए । *1*

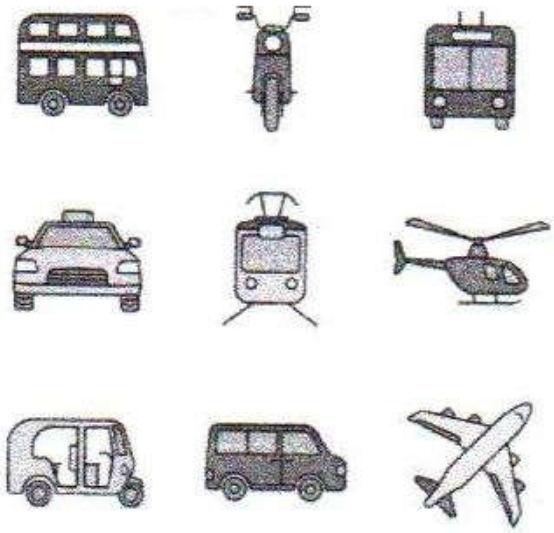
(iii) (क) $P(A)$ ज्ञात कीजिए । *2*

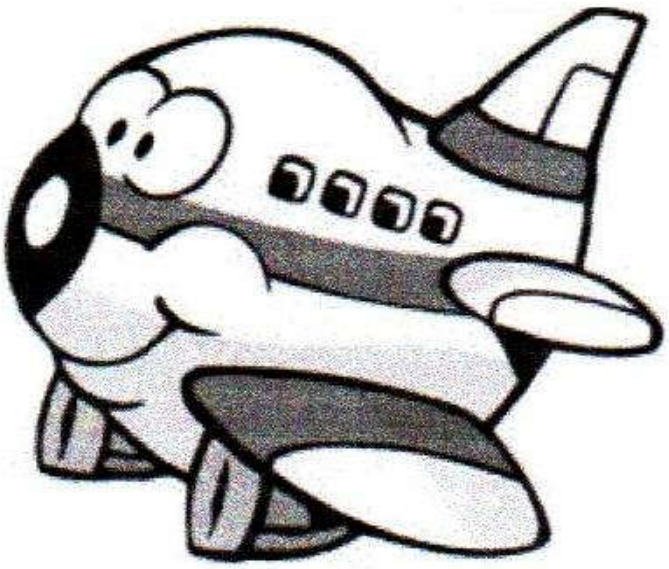
अथवा

(iii) (ख) $P(E_2 \mid A)$ ज्ञात कीजिए । *2*

Case Study – 3

38. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.





Previous records state that the probability of an airplane crash is 0·00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions :

(i) Find the probability that the airplane will not crash. *1*

(ii) Find $P(A \mid E_1) + P(A \mid E_2)$. *1*

(iii) (a) Find P(A). *2*

OR

(iii) (b) Find $P(E_2 \mid A)$. *2*

Marking Scheme
Strictly Confidential
(For Internal and Restricted use only) Senior School Certificate Examination, 2024
MATHEMATICS PAPER CODE 65/2/1

General Instructions:

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totalled up and written in the left- hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without cancelling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**

10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks _____(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totalling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totalling on the title page. • Wrong totalling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) • Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totalling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each	
1.	If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is : (A) 0 (B) 9 (C) 27 (D) 729	
Sol.	(A) 0	1
2.	Let $f : R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is : (A) one-one (B) onto (C) bijective (D) neither one-one nor onto	
Sol.	(C) Bijective	1
3.	If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is : (A) 0 (B) 1 (C) 2 (D) 4	
Sol.	(D) 4	1
4.	The number of points of discontinuity of $f(x) = \begin{cases} \lvert x\rvert + 3, & \text{if } x \le -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \ge 3 \end{cases}$ is : (A) 0 (B) 1 (C) 2 (D) infinite	
Sol.	(B) 1	1

5.	The function $f(x) = x^3 - 3x^2 + 12x - 18$ is : (A) strictly decreasing on R (B) strictly increasing on R (C) neither strictly increasing nor strictly decreasing on R (D) strictly decreasing on $(-\infty, 0)$	
Sol.	(B) Strictly increasing on R	1
6.	$\int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x \cos x} dx$ is equal to : (A) π (B) Zero (0) (C) $\int_0^{\pi/2} \frac{2\sin x}{1+\sin x \cos x} dx$ (D) $\frac{\pi^2}{4}$	
Sol.	(B) Zero (0)	1
7.	The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is : (A) $\cos x - \sin\left(\frac{y}{x}\right)$ (B) $\frac{y}{x}$ (C) $\frac{x^2+y^2}{xy}$ (D) $\cos^2\left(\frac{x}{y}\right)$	
Sol.	(A) $cosx - sin\left(\frac{y}{x}\right)$	1
8.	For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ? (A) $\vec{a}.\vec{b} \geq \lvert\vec{a}\rvert\lvert\vec{b}\rvert$ (B) $\vec{a}.\vec{b} = \lvert\vec{a}\rvert\lvert\vec{b}\rvert$ (C) $\vec{a}.\vec{b} \leq \lvert\vec{a}\rvert\lvert\vec{b}\rvert$ (D) $\vec{a}.\vec{b} < \lvert\vec{a}\rvert\lvert\vec{b}\rvert$	
Sol.	(C) $\vec{a}.\vec{b} \leqslant \lvert\vec{a}\rvert\lvert\vec{b}\rvert$	1

9.	The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by : (A) (1, 0, 0) (B) (2, 0, 0) (C) $(\sqrt{5}, 0, 0)$ (D) (0, 0, 0)	
Sol.	(D) (0, 0, 0)	1
10.	The common region determined by all the constraints of a linear programming problem is called : (A) an unbounded region (B) an optimal region (C) a bounded region (D) a feasible region	
Sol.	(D) a feasible region	1
11.	Let E be an event of a sample space S of an experiment, then $P(S \mid E)$ = (A) $P(S \cap E)$ (B) P(E) (C) 1 (D) 0	
Sol.	(C) 1	1
12.	If $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = i - 3j$, then which of the following is ***false*** ? (A) $a_{11} < 0$ (B) $a_{12} + a_{21} = -6$ (C) $a_{13} > a_{31}$ (D) $a_{31} = 0$	
Sol.	(C) $a_{13} > a_{31}$	1
13.	The derivative of $\tan^{-1}(x^2)$ w.r.t. x is : (A) $\frac{x}{1+x^4}$ (B) $\frac{2x}{1+x^4}$ (C) $-\frac{2x}{1+x^4}$ (D) $\frac{1}{1+x^4}$	
Sol.	(B) $\frac{2x}{1+x^4}$	1

14.	The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')$ is : (A) 1 (B) 2 (C) 3 (D) not defined	
Sol.	(D) not defined	1
15.	The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is : (A) $2\hat{j}$ (B) $\hat{j}$ (C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$	
Sol.	(B) $\hat{j}$	1
16.	Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are : (A) 2, – 1, 6 (B) 2, 1, 6 (C) 2, 1, 3 (D) 2, – 1, 3	
Sol.	(D) 2, -1, 3	1
17.	If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is : (A) 1 (B) 2 (C) 0 (D) – 2	
Sol.	(B) 2	1
18.	If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is : (A) 90° (B) 120° (C) 60° (D) 0°	
Sol.	(A) 90^0	1

	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* For any symmetric matrix A, B′AB is a skew-symmetric matrix. *Reason (R) :* A square matrix P is skew-symmetric if P′ = – P.	
Sol.	(D) Assertion (A) is false, but Reason (R) is true	1
20.	*Assertion (A) :* For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$. *Reason (R) :* For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.	
Sol.	(C) Assertion (A) is true, but Reason (R) is false.	1
	SECTION B	
21.	(a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.	
Sol.	The given expression $= \frac{-\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4}$ $= \frac{-\pi}{12}$	$1\frac{1}{2}$ $\frac{1}{2}$
	OR	
	(b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.	
Sol.	$-1 \leqslant x^2 - 4 \leqslant 1$ $\Rightarrow 3 \leqslant x^2 \leqslant 5$ Domain $= [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$	$\frac{1}{2}$ $\frac{1}{2}$

	Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	1
22.	(a) If f(x) = $\lvert \tan 2x \rvert$, then find the value of f′(x) at x = $\frac{\pi}{3}$.	
Sol.	$f(x) = -tan2x,\ \frac{\pi}{4} < x < \frac{\pi}{2}$ $f'(x) = -2sec^2 2x,\ \frac{\pi}{4} < x < \frac{\pi}{2}$ $f'(\frac{\pi}{3}) = -2(-2)^2 = -8$	$\frac{1}{2}$ 1 $\frac{1}{2}$
	OR	
	(b) If y = cosec(cot^{-1} x), then prove that $\sqrt{1+x^2}\,\frac{dy}{dx} - x = 0$.	
Sol.	$y = \sqrt{1 + cot^2(cot^{-1}x)} = \sqrt{1+x^2}$ $\Rightarrow \frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$ $\Rightarrow \sqrt{1+x^2}\frac{dy}{dx} - x = 0$	$\frac{1}{2}$ 1 $\frac{1}{2}$
23.	If M and m denote the local maximum and local minimum values of the function f(x) = x + $\frac{1}{x}$ (x ≠ 0) respectively, find the value of (M – m).	
Sol.	$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2} = \frac{(x+1)(x-1)}{x^2}$ $f'(x) = 0 \Rightarrow x = -1,\ 1$ $f''(x) = \frac{2}{x^3} \Rightarrow f''(-1) = -2 < 0$ ∴ -1 is a point of local maximum The local maximum value = $f(-1) = -2 = M$ $f''(1) = 2 > 0$ ∴ 1 is point of local minimum The local minimum value = $f(1) = 2 = m$ $M - m = -4$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

24.	**Find :** $\int \frac{e^{4x}-1}{e^{4x}+1}\,dx$	
Sol.	Given integral $=\int \frac{e^{4x}-1}{e^{4x}+1}dx$ $=\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx$ $=\frac{1}{2}\int \frac{2(e^{2x}-e^{-2x})}{e^{2x}+e^{-2x}}dx$ $=\frac{1}{2}log\lvert e^{2x}+e^{-2x}\rvert + C$	$\frac{1}{2}$ $\frac{1}{2}$ 1
25.	Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.	
Sol.	$f'(x) = e^x + e^{-x} + 1 - \frac{1}{1+x^2}$ $=e^x + \frac{1}{e^x} + \frac{x^2}{1+x^2} > 0\ for\ all\ x \in R$ $\therefore$ f is strictly increasing over its domain R	1 1
	Section C	
26.	(a) If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.	
Sol	$\frac{dx}{dt} = e^{cos3t} \times (-sin3t) \times 3$ $\frac{dy}{dt} = e^{sin3t} \times (cos3t) \times 3$ $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^{sin3t} \times (cos3t)}{-e^{cos3t} \times (sin3t)}$ $x = e^{cos3t} \Longrightarrow cos3t = logx$ $y = e^{sin3t} \Longrightarrow sin3t = logy$ $\therefore \frac{dy}{dx} = \frac{-ylogx}{xlogy}$	1 1 1

	OR	
	(b) Show that : $\frac{d}{dx}(\lvert x \rvert) = \frac{x}{\lvert x \rvert}, x \neq 0$	
Sol.	$\frac{d(\lvert x \rvert)}{dx} = \frac{d(\sqrt{x^2})}{dx}, x \neq 0$ $= \frac{1}{2}(x^2)^{-\frac{1}{2}} \times \frac{d(x^2)}{dx}$ $= \frac{1}{2\sqrt{x^2}} 2x = \frac{x}{\lvert x \rvert}$	1 $1\frac{1}{2}$ $\frac{1}{2}$
27.	(a) Evaluate : $\int_{-2}^{2} \sqrt{\frac{2-x}{2+x}}\, dx$	
Sol.	(a) $\int_{-2}^{2} \sqrt{\frac{2-x}{2+x}}\, dx$ $= \int_{-2}^{2} \frac{2-x}{\sqrt{4-x^2}}\, dx$ $= \int_{-2}^{2} \frac{2}{\sqrt{4-x^2}}\, dx - \int_{-2}^{2} \frac{x}{\sqrt{4-x^2}}\, dx$ $= 2\int_{0}^{2} \frac{2}{\sqrt{4-x^2}}\, dx - 0$ [$\frac{2}{\sqrt{4-x^2}}$ is even, $\frac{x}{\sqrt{4-x^2}}$ is odd] $= 4\int_{0}^{2} \frac{1}{\sqrt{4-x^2}}\, dx$ $= 4\, sin^{-1}\frac{x}{2}\Big\rvert_{0}^{2}$ $= 2\pi$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$
	OR	
	(b) Find : $\int \frac{1}{x\,[(\log x)^2 - 3\log x - 4]}\, dx$	
Sol.	Let $logx = t \Rightarrow \frac{1}{x}dx = dt$ The given integral becomes $= \int \frac{1}{t^2-3t-4}\, dt$ $= \int \frac{1}{\left(t-\frac{3}{2}\right)^2 - \left(\frac{5}{2}\right)^2}\, dx$	$\frac{1}{2}$ $\frac{1}{2}$ 1

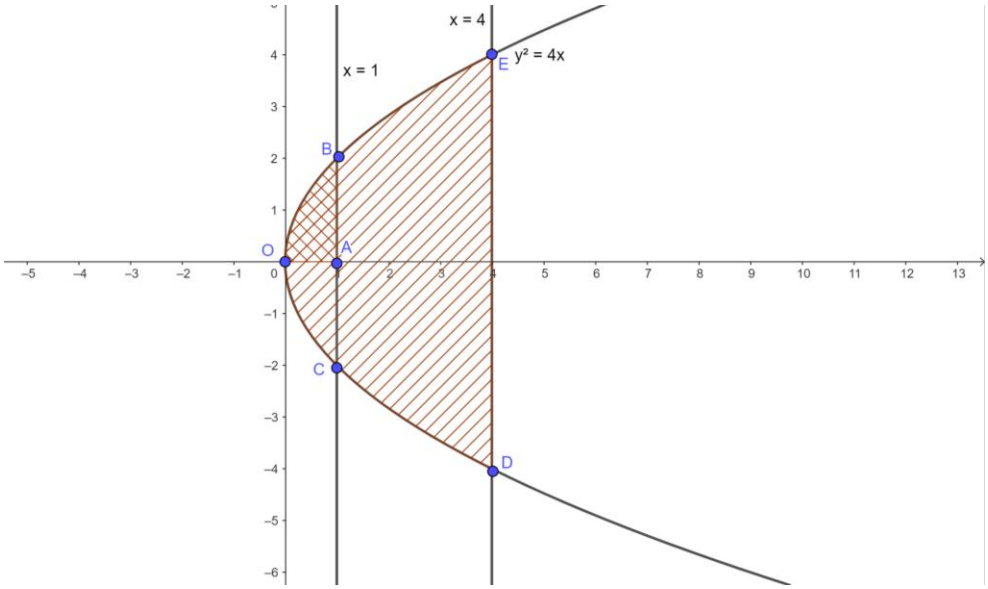
	$= \frac{1}{5} log \left\lvert \frac{t-4}{t+1} \right\rvert + C$ $= \frac{1}{5} log \left\lvert \frac{logx-4}{logx+1} \right\rvert + C$	$\frac{1}{2}$ $\frac{1}{2}$
28.	(a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$; y = 2, when x = 1.	
Sol.	Given differential equation can be written as $\frac{dy}{dx} = \frac{2xy+y^2}{2x^2} = \frac{y}{x} + \frac{y^2}{2x^2}$ Let $y = vx \Longrightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ The equation becomes $x\frac{dv}{dx} = \frac{1}{2}v^2$ $\Longrightarrow \frac{dv}{v^2} = \frac{1}{2} \times \frac{dx}{x}$ Integrating both sides, we get $\frac{-1}{v} = \frac{1}{2} log\lvert x\rvert + C$ $\Longrightarrow -\frac{x}{y} = \frac{1}{2} log\lvert x\rvert + C$ $y = 2, x = 1$ gives $C = -\frac{1}{2}$ The particular solution is $-\frac{x}{y} = \frac{1}{2} log\lvert x\rvert - \frac{1}{2}$ $or, y = \frac{2x}{1 - log\lvert x\rvert}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$
	OR	
	(b) Find the general solution of the differential equation : $y\,dx = (x + 2y^2)\,dy$	
Sol.	Given differential equation can be written as $\frac{dx}{dy} - \frac{x}{y} = 2y$ Integrating Factor = $e^{\int \frac{-1}{y} dy} = \frac{1}{y}$ Solution is $x\frac{1}{y} = \int 2dy$ $\Longrightarrow \frac{x}{y} = 2y + C$ $\Longrightarrow x = 2y^2 + Cy$	1 1 $\frac{1}{2}$ $\frac{1}{2}$

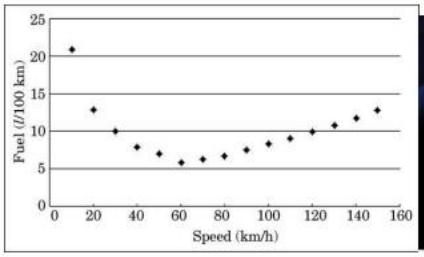
29.	The position vectors of vertices of Δ ABC are A($2\hat{i} - \hat{j} + \hat{k}$), B($\hat{i} - 3\hat{j} - 5\hat{k}$) and C($3\hat{i} - 4\hat{j} - 4\hat{k}$). Find all the angles of Δ ABC.	
Sol.	$cosA = \frac{\overrightarrow{AB}.\overrightarrow{AC}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AC}\rvert} = \frac{(-\hat{i} - 2\hat{j} - 6\hat{k}).(\hat{i} - 3\hat{j} - 5\hat{k})}{\sqrt{41}\sqrt{35}} = \frac{35}{\sqrt{41}\sqrt{35}} = \frac{\sqrt{35}}{\sqrt{41}}$ $A = cos^{-1}\left(\frac{\sqrt{35}}{\sqrt{41}}\right)$ $cosB = \frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert} = \frac{(\hat{i} + 2\hat{j} + 6\hat{k}).(2\hat{i} - \hat{j} + \hat{k})}{\sqrt{41}\sqrt{6}} = \frac{6}{\sqrt{41}\sqrt{6}} = \frac{\sqrt{6}}{\sqrt{41}}$ $B = cos^{-1}\left(\frac{\sqrt{6}}{\sqrt{41}}\right)$ $cosC = \frac{\overrightarrow{CB}.\overrightarrow{CA}}{\lvert\overrightarrow{CB}\rvert\lvert\overrightarrow{CA}\rvert} = \frac{(-2\hat{i} + \hat{j} - \hat{k}).(-\hat{i} + 3\hat{j} + 5\hat{k})}{\lvert\overrightarrow{CB}\rvert\lvert\overrightarrow{CA}\rvert} = 0$ cos C = 0 $\Rightarrow C = \frac{\pi}{2}$	1 1 1 1
30.	A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.	
Sol.	<table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>P(X)</td><td>$\frac{6}{36} = \frac{1}{6}$</td><td>$\frac{10}{36} = \frac{5}{18}$</td><td>$\frac{8}{36} = \frac{2}{9}$</td><td>$\frac{6}{36} = \frac{1}{6}$</td><td>$\frac{4}{36} = \frac{1}{9}$</td><td>$\frac{2}{36} = \frac{1}{18}$</td></tr></table>	$\frac{1}{2} \times 6 = 3$
31.	Find : $\int x^2 \cdot \sin^{-1}(x^{3/2})\, dx$	
Sol.	Let $x^{\frac{3}{2}} = t$ $\Rightarrow \frac{3}{2}x^{\frac{1}{2}}dx = dt$ The given integral becomes $\frac{2}{3}\int t\, sin^{-1}t dt$ $= \frac{2}{3}\left[sin^{-1}t \times \frac{t^2}{2} - \int \frac{1}{\sqrt{1-t^2}} \times \frac{t^2}{2} dt\right]$ $= \frac{1}{3}\left[sin^{-1}t \times t^2 + \int \frac{1-t^2-1}{\sqrt{1-t^2}} dt\right]$ $= \frac{1}{3}\left[sin^{-1}t \times t^2 + \int \sqrt{1-t^2}\, dt - \int \frac{1}{\sqrt{1-t^2}} dt\right]$ $= \frac{1}{3}\left[t^2 sin^{-1}t + \frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}sin^{-1}t - sin^{-1}t\right]$+C	$\frac{1}{2}$ $\frac{1}{2}$ 1

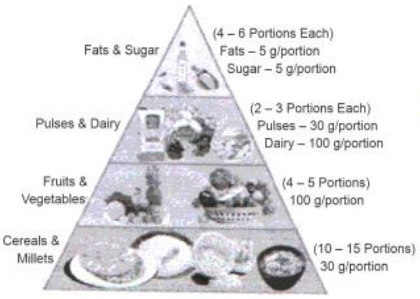
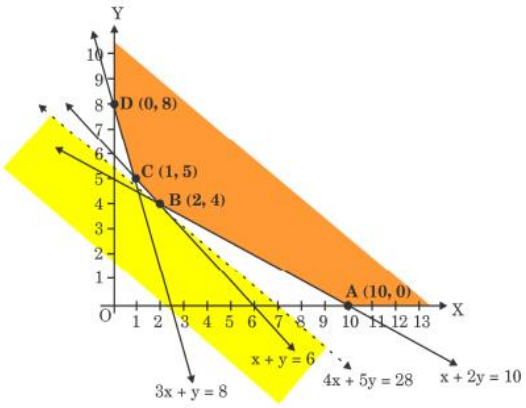
	$=\frac{1}{3}\left[t^2 sin^{-1}t + \frac{t}{2}\sqrt{1-t^2} - \frac{1}{2}sin^{-1}t\right] + C$ $= \frac{1}{3}\left[x^3 sin^{-1}\left(x^{\frac{3}{2}}\right) + \frac{x^{\frac{3}{2}}}{2}\sqrt{1-x^3} - \frac{1}{2}sin^{-1}\left(x^{\frac{3}{2}}\right)\right] + C$	1
	Section D	
32.	(a) Show that a function f : R → R defined by f(x) = $\frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function f : R → A becomes an onto function.	
Sol.	Let $f(x_1) = f(x_2)\ for\ some\ x_1, x_2 \in R$ Then $\frac{2x_1}{1+{x_1}^2} = \frac{2x_2}{1+{x_2}^2}$ $\Rightarrow x_1 + x_1{x_2}^2 = x_2 + {x_1}^2 x_2$ $\Rightarrow (x_1 - x_2) - x_1x_2(x_1 - x_2) = 0$ $\Rightarrow (x_1 - x_2)(1 - x_1x_2) = 0$ $\Rightarrow x_1 - x_2 = 0$ or $1 - x_1x_2 = 0$ $\Rightarrow x_1 = x_2$ or $x_1x_2 = 1$, so if $x_1x_2 = 1, x_1 \neq x_2$ Hence f is not one -one Let y = f(x) where $x \in R$ Then $y = \frac{2x}{1+x^2}$. Here, for x = 0, y = 0 If $y \neq 0$, then $y = \frac{2x}{1+x^2}$ $\Rightarrow yx^2 - 2x + y = 0$ $\Rightarrow x = \frac{2 \pm \sqrt{4-4y^2}}{2y}$ For x to be real, $4 - 4y^2 \geq 0$ $\Rightarrow y^2 \leq 1$ $\Rightarrow -1 \leq y \leq 1$ Hence, range = $[-1,1] \neq codomain$ Hence, f is not onto. For the given function to become onto, $A = [-1,1]$	2 2 1

	OR	
	(b) A relation R is defined on N × N (where N is the set of natural numbers) as : (a, b) R (c, d) ⇔ a – c = b – d Show that R is an equivalence relation.	
Sol.	Let $(a, b) \in N \times N$ We have $a - a = b - b$ This implies that (a, b) R (a, b) $\forall (a, b) \in N \times N$ Hence R is reflexive Let (a, b) R (c, d) for some $(a, b), (c, d) \in N \times N$ Then $a - c = b - d$ $\Rightarrow c - a = d - b$ $\Rightarrow$(c, d) R (a, b) Hence, R is symmetric. Let (a, b) R (c, d), (c, d) R (e, f) for some $(a, b), (c, d), (e, f) \in N \times N$ Then $a - c = b - d, c - e = d - f$ $\Rightarrow a - c + c - e = b - d + d - f$ $\Rightarrow a - e = b - f$ $\Rightarrow$(a, b) R (e, f) Hence, R is transitive Thus, R is an equivalence relation.	$1\frac{1}{2}$ $1\frac{1}{2}$ 2
33.	Find the equation of the line which bisects the line segment joining points A(2, 3, 4) and B(4, 5, 8) and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.	
Sol	Let direction ratios of the required line be a, b, c. $\because$ the required line is perpendicular to both the given lines $\therefore 3a - 16b + 7c = 0$ and $3a + 8b - 5c = 0$ $\Rightarrow \frac{a}{24} = \frac{b}{36} = \frac{c}{72}$ $\Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{6}$ The mid-point of the line-segment AB is (3, 4, 6) Hence, the required equation of the line is $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-6}{6}$	1 $\frac{1}{2}$ $1\frac{1}{2}$ 1 1

34.	(a) Solve the following system of equations, using matrices : $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$ where x, y, z ≠ 0	
Sol.	Given system of linear equations is equivalent to AX = B, where $A=\begin{bmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{bmatrix}, X=\begin{bmatrix}\frac{1}{x}\\ \frac{1}{y}\\ \frac{1}{z}\end{bmatrix}, B=\begin{bmatrix}4\\1\\2\end{bmatrix}$ $\frac{1}{2}$ $\|A\|=1200\neq 0$ $\frac{1}{2}$ Cofactors of the elements of A are $A_{11}=75,\ A_{12}=110,\ A_{13}=72$ $A_{21}=150,\ A_{22}=-100,\ A_{23}=0$ $A_{31}=75,\ A_{32}=30,\ A_{33}=-24$ $adjA=\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$ 2 $A^{-1}=\frac{adjA}{\|A\|}=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$ $\frac{1}{2}$ $X=A^{-1}B=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}\begin{bmatrix}4\\1\\2\end{bmatrix}$ $=\frac{1}{1200}\begin{bmatrix}600\\400\\240\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{5}\end{bmatrix}$ 1 $\therefore x=2,\ y=3,\ z=5$ $\frac{1}{2}$	
	OR	
	(b) If $A=\begin{bmatrix}1 & \cot x\\ -\cot x & 1\end{bmatrix}$, show that $A'A^{-1}=\begin{bmatrix}-\cos 2x & -\sin 2x\\ \sin 2x & -\cos 2x\end{bmatrix}$.	
Sol.	$\|A\|=1+cot^2x=cosec^2x$ $adjA=\begin{bmatrix}1 & -cotx\\ cotx & 1\end{bmatrix}$ $\frac{1}{2}$ $1\frac{1}{2}$	

	$A^{-1} = \frac{adjA}{\|A\|} = \frac{1}{cosec^2x}\begin{bmatrix} 1 & -cotx \\ cotx & 1 \end{bmatrix}$ $A' = \begin{bmatrix} 1 & -cotx \\ cotx & 1 \end{bmatrix}$ $A'A^{-1} = \frac{1}{cosec^2x}\begin{bmatrix} 1-cot^2x & -2cotx \\ 2cotx & 1-cot^2x \end{bmatrix}$ $= \begin{bmatrix} sin^2x - cos^2x & -2sinxcosx \\ 2sinxcosx & sin^2x - cos^2x \end{bmatrix}$ $= \begin{bmatrix} -cos2x & -sin2x \\ sin2x & -cos2x \end{bmatrix}$	1 $\frac{1}{2}$ $1\frac{1}{2}$
35.	If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.	
Sol.	 A_1 = Area (region OABO) = $\int_0^1 2\sqrt{x}\,dx = 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \frac{4}{3}$ A_2 = Area (region ODEO) = $2\int_0^4 2\sqrt{x}\,dx = 4 \times \frac{2}{3}[2^3] = \frac{64}{3}$ $A_1 : A_2 = \frac{4}{3} : \frac{64}{3} = 1:16$	2 1 1 1

	Section E	
36.	Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.   The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$. On the basis of the above information, answer the following questions : (i) Find F, when V = 40 km/h. (ii) Find $\frac{dF}{dV}$. (iii) (a) Find the speed V for which fuel consumption F is minimum. **OR** (iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0{\cdot}01$.	
Sol.	(i) When V = 40 km/h, F = 36/5 ℓ/100km (ii) $\frac{dF}{dV} = \frac{V}{250} - \frac{1}{4}$ (iii)(a) $\frac{dF}{dV} = 0$ $\Longrightarrow$V = 62.5 km/h $\frac{d^2F}{dV^2} = \frac{1}{250} > 0$ at V = 62.5 km/h Hence, F is minimum when V = 62.5 km/h	1 1 1 1 $\frac{1}{2}$ $\frac{1}{2}$

	OR	
	(iii) (b) $\frac{dF}{dV} = -0.01$ $\Rightarrow \frac{V}{250} - \frac{1}{4} = \frac{-1}{100}$ $\Rightarrow V = 60$ km/h $F = \frac{60^2}{500} - \frac{60}{4} + 14 = 6.2\,\ell/100km$ Quantity of fuel required for 600 km $= 6.2 \times 6 = 37.2\,\ell$	1 $\frac{1}{2}$ $\frac{1}{2}$
37.	The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.  Figure-1  Figure-2 A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2. On the basis of the above information, answer the following questions : (i) Identify and write all the constraints which determine the given feasible region in Figure-2. (ii) If the objective is to minimize cost Z = 16x + 20y, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region.	

Sol.

(i)Constraints are $x + 2y \geq 10$ — $1\frac{1}{2}$

$x + y \geq 6$

$3x + y \geq 8$

$x \geq 0$

$y \geq 0$ — $\frac{1}{2}$

(ii)

Corner points	Value of Z = 16x + 20y
A (10, 0)	160
B (2, 4)	112
C (1, 5)	116
D (0, 8)	160

$1\frac{1}{2}$

The minimum cost is ₹112 — $\frac{1}{2}$

38.

Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.





Previous records state that the probability of an airplane crash is 0·00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

	Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey. On the basis of the above information, answer the following questions : (i) Find the probability that the airplane will not crash. (ii) Find $P(A \mid E_1) + P(A \mid E_2)$. (iii) (a) Find P(A). **OR** (iii) (b) Find $P(E_2 \mid A)$.	
Sol.	(i) $P(E_2) = 1 - 0.0000001$ $= 0.9999999$ (ii) $P(A/E_1) + P(A/E_2) = \frac{95}{100} + 1 = \frac{195}{100}$	1 1
	(iii)(a) $P(A) = P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)$ $= \frac{1}{10000000} \times \frac{95}{100} + \frac{9999999}{10000000} \times 1$ $= \frac{95+999999900}{1000000000} = \frac{999999995}{1000000000}$	1 1
	OR	
	(iii)(b) $P(E_2/A) = \frac{P(E_2) \times P(A/E_2)}{P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2)}$ $= \frac{\frac{9999999}{10000000}}{\frac{999999995}{1000000000}} = \frac{999999900}{999999995}$	1 1